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Progressive Lifting of Shallow Sewers Due to Frost Heave Actions: Investigation of a Lumped Parameter Frost Heave Model

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The objective of this report is to complete work undertaken at the University Minnesota in the mid 1990's. This work was directed at developing an understanding of frost heaves in relationship to the lifting of buried objects such as shallow sewers.

The critical findings in the field investigation were detailed in a 1995 final report by Ray Sterling, "Progressive Lifting of Shallow Sewers: Field Investigation." Preliminary details of the modeling work were provided in an interim project report and complete details were provided in the MS Thesis of Lingjun Hou, "An Investigation of a lumped Parameter Frost Heave Model" published by the University of Minnesota in January 1994.

The objective of the current report is to outline the critical findings in the modeling work detailed in the previous interim report and Hou's thesis. This document should be seen as a compliment to the original final report submitted by Sterling in 1995. The central objective of the modeling study is the investigation of the lumped parameter frost heave model developed by Blanchard and Fremond.

The work in this report shows that a lumped porosity model can generates an accurate comparison with experiments and is a feasible tool for the investigation of the effect of freeze thaw on buried infrastructure.

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Progressive Lifting of Shallow Sewers Due to Frost Heave Actions: Investigation of a Lumped Parameter Frost Heave Model

Final Report

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EXECUTIVE SUMMARY

The objective of this report is to complete work undertaken at the University Minnesota in the mid 1990's. This work was directed at developing an understanding of frost heave in relationship to the lifting of buried objects such as shallow sewers.

There were three distinct parts to this work.

1. A field Investigation where the progressive lifting of a sewer in Little Canada over the winter of 1994 was measured.
2. The development and testing of a frost heave model.
3. The development of a thermal thawing model suitable for describing the movement of buried structures during freeze-thaw cycles.

The critical findings in the field investigation were detailed in a Final Report:

"Progressive Lifting of Shallow Sewers: Field Investigation," Prepared by Ray Sterling in October 1995.

Preliminary details of the modeling work were provided in an interim project report and complete details were provided in the MS Thesis of Lingjun Hou,"An Investigation of a Lumped Parameter Frost Heave Model," published by the University of Minnesota in January 1994.

The objective of the current report is to outline the critical findings in the modeling work detailed in the previous interim report and Hou's thesis. This document should be seen as a compliment to the original final report submitted by Sterling in 1995. The central objective of the modeling study is the investigation of the lumped parameter frost heave model developed by Blanchard and Fremond (1). The key contributions are:

1. The development of general transport governing equations based on an averaging multi-phase analysis.

2. The application of an efficient numerical phase change algorithm for the solution of the lumped parameter frost heave model. In this numerical solution, the heaving deformations of the domain and convection effects are fully accounted for.
3. The development of a simplified two-dimensional thermal model that can be used to investigate the effects of freeze-thaw cycles on buried infrastructure.

The work in this report shows that a lumped porosity model can generate an accurate comparison with experiments and is a feasible tool for the investigation of the effect of freeze thaw on buried infrastructure. It is recommended that further work is undertaken to develop user friendly software based around the lumped porosity model and extend its application across a range of transportation infrastructure problems. In the mean time it is recommended that practical construction methods and non frost susceptible materials are utilized, see Svec and Gallagher(2). A synthesis of innovative construction practices should also be completed.

CHAPTER 1

INTRODUCTION

Introduction to Frost Heave Phenomena

Under winter conditions, when water freezes and expands in a soil such as clay or silt, a phenomenon known as frost heave can occur. The expansion of the freezing zone in the ground results in the upward movement of structures. In spring, as the ground thaws, an excess of water is produced, this excess reduces the effective stress in the soil and results in a loss of bearing capacity.

The critical feature to note in frost heaving is that, in most cases, the level of heaving can not be fully accounted for by simply considering the freezing expansion of the existing water in the soil. Experiments carried out by Taber (3 and 4) showed that when a soil column freezes additional moisture is sucked into the freezing zone thereby increasing the level of heave. Taber conducted experiments in which the water in the soil was replaced by liquids (benzene and nitrobenzene), which decrease in volume when frozen. In these experiments, provided the freezing soil column was able to draw up additional liquid into the freezing zone, significant heaving was still observed.

There are several factors, which influence the moisture migration and frost heaving in freezing soils:

1. Capillary size: the size of the capillaries in the soil determines the height to which water may be lifted above the phreatic surface (water table) by surface tension. In the absence of other effects, the water rise is inversely proportional to the diameter of the soil capillaries. This relationship, however, can break down in fine soils where the presence of colloids decreases the apparent permeability.
2. Size of soil particle: the size of the soil particles has a large influence on the level of heave. When a growing ice crystal closely approaches a soil particle the water separating them is reduced to a thin film and further growth of the crystal can only take place as molecules of water enter this film. If the soil particle is small and the crystal growth slow, water molecules have time to enter between the ice and the particle and the growing crystal will exclude the particle. This process, referred to as segregation, leads to the formation of ice lenses in the soil that

significantly contributes to the level of heave. On the other hand, if the soil particles are large and freezing is rapid, water can not freely move into the layer between the ice crystals, the particle is gradually surrounded by the ice and the heave associated with segregation is reduced.

3. Water-content of soil: the amount of heaving that takes place when soils freeze is limited by the supply of available water; not only the water present in soils before freezing, but also the water that can be drawn from points below the depth of freezing. So if the water table is close to the frost front in the soil, there is an increased likelihood that water can be drawn into the freezing zone. Since there is considerable friction resistance to water passage through the soil, frost heaving tends to decrease as the depth of the water table increases. However, the effect of a low water table could be in part compensated by changes in other parameters such as a slower cooling rate.
4. Rate of cooling: it is evident that, over a certain range of temperatures, increasing the rate of cooling by applying lower cold-side temperatures does increase the heave rate (5). The rate of cooling is also dependent on the thermal conductivities and specific heats of the different materials present and by the latent heat effects present in the phase change. Most soil minerals differ from one another little in their thermal properties; however, they differ significantly from water and ice. At temperatures above freezing, the rate of cooling of soils is affected greatly by the porosity and the water content. When the soil temperature drops below freezing, the fraction of water converted into ice at a given temperature will also influences the rate of cooling.
5. External pressure: when vertical displacement is prevented in a frost-heaving soil a positive pressure is developed at the ice-water interface. This pressure is known as the heaving pressure (5). Such pressures can be sufficiently high to destroy foundations and lift buildings. Determination of the maximum force exerted by growing ice crystals is an interesting but difficult problem. It is common to state that, to prevent heaving, the applied external pressure should be equal to the pressure generated by ice formation. The original experiment by Taber (4), however, showed that the force exerted by the ice crystals is greater than expected, because when forming each layer of ice it is necessary to pry apart the clay cylinder used in the test and

overcome side wall friction. It is clear from the tests that excessive heaving still may occur under surface loads of 150 or 200 lb/in^2, but that the rate of heaving is smaller than that under lower applied pressure. On the other hand, when the surface pressure is increased, the freezing point of water in the soil will be depressed. So, even though extremely high pressures would compact the clay, reduce porosity, and make it less permeable to water, the entrance of water would be only retarded rather than prevented, and heaving can still take place.

6. Effects of thawing and refreezing: it has been shown that freezing may result in the concentration of excessive amounts of water in the form of ice at, or near, the surface. During or immediately after thawing, when an excessive amount of water is left in the surface soil, refreezing should result in greater segregation than on the first freezing, since segregation is accentuated by high water content near the soil surface.

Through the above discussion, the complexity of frost heave problems can be appreciated. There are, however, three simple conditions that are required for frost heaving to occur:

1. There must be freezing temperature conditions within the soil.
2. There must be a source of water to feed the ice lens growth.
3. The physical composition of the soil must allow sufficient migration of the moisture to the freezing front to cause heaving within the time scale of the propagation of the freezing front into the soil.

Problems Caused by Frost Heave

When the air temperature falls below freezing for extended periods of time it is possible for the pore water in soils to freeze. Frost action in soils result in several important engineering consequences. First, the volume of the water in the pore spaces can immediately increase about 10% just due to the volumetric expansion of water upon freezing. A second but significantly more important factor is the formation of ice crystals and lenses in the soil. These lenses can grow to several centimeters in thickness and cause heaving and damage to light surface structures such as small buildings and highway pavements. The ice lenses can be seen as the dark bands within the soil.

If soils simply freeze and expand uniformly, structures would be evenly displaced since the pressures associated with ice lens formation are much greater than the loading provided by these light structures. However, just as with swelling and shrinking soils, the volume change is usually uneven, causing differential movement and thus structural damage.

Also, during the spring, the ice lenses melt and greatly increase the water content and decrease the strength of the soil. This thaw weakening of soils allows rapid damage to road pavements under heavy loads, potholes occur easily and fine soils can be "pumped" through cracks in the road pavement. Damage to highways in cold regions because of frost action is a very expensive problem and highway load restrictions during the thaw period add to the economic burden.

In addition to the load restrictions mentioned above, other techniques used to combat frost heave and thaw weakening problems include lowering of the ground water table, removal of frost susceptible soils in the subgrade or foundation (2), use of impervious membranes, chemical additives, and foam insulation under highways, buildings, and railroads, etc.

The refinements of these techniques for reducing or eliminating frost-related problems is aided by an ability to simulate the soil conditions throughout the winter and to refine solutions without resorting to full-scale testing. Improvements in our ability to simulate frost heaving and thaw weakening in soils are the ultimate goals of the research described here.

Objective and Scope

The objectives of the research are:

1. To develop an effective lumped macro-scale frost heaving model: most existing models deal with micro-scale process where an attempt is made to predict the moisture migration and velocity of ice lens, the 'Rigid Ice Model' by O'Neill and Miller (6) is an example of such a model. Since it is very hard to deal with micro-scale process, the alternative engineering approach is to develop a lumped parameter model in which it is attempted to model the macro effects of the

micro-scale processes rather than the processes themselves. Furthermore, the lumped parameter model has considerable advantages with its simplicity for application to two- or three-dimensional problems.

2. To develop an efficient numerical algorithm based on the enthalpy approach (7): the key feature in a phase change problem is the presence of a moving boundary on which heat and mass balance conditions have to be met. The classical approach to deal with moving boundary problem is the Stefan approach (8). A popular approach in the numerical modeling of such problems is the so-called enthalpy method, see Voller and Cross (7). The major advantage of this method is that despite the presence of moving boundaries, the problem is cast in a conservative form that allows for the solution to be obtained on a fixed space grid. Also, since the water content-temperature curve has a discontinuity when ice first forms, there are problems in numerical calculation. An efficient algorithm will be discussed in this report.

CHAPTER 2.

THE MECHANISM OF FROST HEAVE

In chapter 1, the frost heave problem was briefly introduced. Here the mechanism of frost heave will be described in greater detail.

The phenomenon of frost heave may be considered to consist of four major rate processes: heat transport, water transport, ice growth, and soil deformation (9). A one-dimensional experiment scheme by Miller (10) is shown in Fig 2.1. Soil is confined in a rigid cylinder with a porous base at the bottom and a loaded piston at the top to simulate an overburden pressure. A temperature gradient is established across the soil sample such that the temperature of the soil decreases with elevation over time from an initial constant temperature condition.

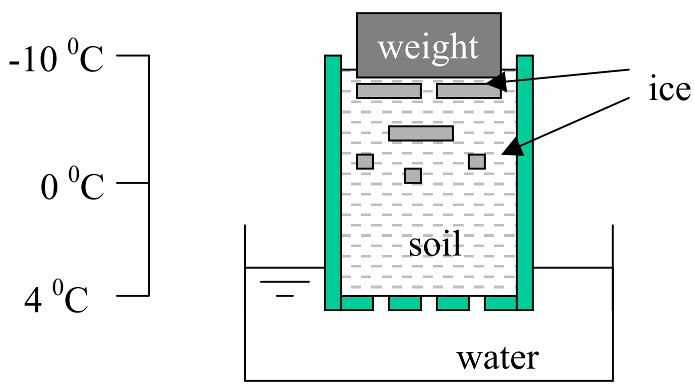


Figure 2.1 Schematic of a Frost heave Test (After Miller 1978)

Moisture Migration

At the beginning of the experiment, pore water near the top of the soil sample will freeze. As discussed in chapter 1, frost heaving requires additional moisture to migrate to the freezing zone. In saturated soils, migration occurs only in the form of water while in unsaturated soils, vapor diffusion is also possible. Evidence presented by Nakano et. al. (11,12), however, suggests that the vapor diffusivity in frozen soil is very low. This is expected because when pore ice is formed it inhibits vapor flow but as will be discussed later, there is an adsorbed water film between pore ice and soil particles through which some movement of water can occur. Also, from the magnitude of the respective diffusivities, Williams and Smith (13) have shown that the vapor diffusion in frozen soil is several orders of magnitudes less than the water flow.

The classic works of Taber (3) and Beskow (14) on the migration of water to a growing ice lens represented, until the 1950s, the most serious attempts to identify the mechanism of frost heaving. Taber (3, 4) attributed the migration to "molecular cohesion" and identified the factors controlling ice segregation as soil particle size, amount of water available, size of voids, void ratio, and rate of cooling. Beskow (14) related the suction pressure to "capillary rise" and showed the relationship of the height of capillary rise to grain size and depth to the water table. Neither of these explanations provided a rigorous theory for frost heave.

From the 1960s to the 1980s, three fundamentally different explanations for ice segregation and frost heave have received considerable attention. These are capillary theory, secondary heaving theory and adsorption force theory. In a broad sense these three theories are in general agreement on the major factors affecting frost heave (15). Each theory is described in the following sections.

Capillary Theory

In general explanations of frost heave phenomena, capillary theory (or the surface tension effect) are often used to explain the large suctions and the consequent moisture migration into freezing soil. The philosophy of the capillary theory is that the heave pressure and the suction pressures that develop during the formation of ice lenses are related to the porous matrix of the soil (15). Penner (16) and Gold (17) observed that the geometry of the porous soil matrix in which ice lenses develop is very important in determining the magnitude of the suction. Penner concluded that moisture tensions develop as a result of freezing point depressions and that higher tensions develop in soils with small pores than in soils with large pores because the freezing point decreases with the radius of curvature of the ice/water interface.

The mathematical form of the capillary theory is

$$P_i = P_w + \frac{2\sigma_{iw}}{r_{iw}} \tag{2.1}$$

where:

P_i is steady state heaving pressure at the base of the ice lens (or pore ice pressure).

P_w is pore water pressure.

σ_{iw} is surface tension at an ice/water interface

r_{iw} is radius of the ice/water interface.

It is assumed that adsorption forces are negligible and that the soil is an ideal granular material (15). To determine the maximum heaving pressure, the radius, $r_{i,w}$, is set to the radius of the pore necks through which ice must grow.

Penner (18) used experiments to verify Eq.(2.1) for uniform glass spheres in a close-packed array. For nongranular soil, however, it is hard to choose a representative value of $r_{i,w}$. Also Penner (19) found that the heaving pressure from Eq.(2.1) is too large when the average value of $r_{i,w}$ is used, but that the results agree well with the measured values when the size of the smallest particles is used (15).

Because the effective stress beneath or between ice lenses will increase when frost heave occurs, the rate of heaving for a given soil also depends on the compressibility of the soil in addition to the rate of heat extraction at the freezing front, the stress borne by the ice lens, the suction in the pore water and the hydraulic conductivity of the zone beneath the ice lens formation. Bishop and Blight (20) and Miller (10) suggested applying Terzaghi's concept of effective stress and pore pressure for frozen soils. This effective stress can be expressed as:

$$\sigma_s = \sigma - \sigma_n \tag{2.2}$$

where:

σ_s is the effective inter-granular stress

σ is the total stress right below ice lens

σ_n is the neutral stress (or pressure supported by the pore contents)

In saturated soils, σ_n is equal to the pore water pressure P_w (15). Because σ_n is always negative beneath a growing ice lens, the effective stress on the soil beneath is always higher than before freezing. If the soil is practically incompressible under this stress (like most dense sands), then σ_n has little effect on the soil structure. But, if the soil is compressible (e.g. clay soils), then the void ratio decreases as the effective stress increases and the soil becomes more dense. This effect leads to two important influences on frost heave: 1) the compensating effect of the void ratio decreases beneath the ice lens may lessen or eliminate the apparent frost heave. The resulting effect is to over consolidate the soil by freezing, Nixon and Morgensten (21), and Chamberlain and Blouin (22) because pore size decreases due to the increase of effective stress, the pore water suction, $-P_w$, and the frost heaving stress also will increase, and this will affect the hydraulic conductivity of the soil.

In general, according to the capillary theory, the frost heave should depend on 1) the rate of heat removal, 2) the pore size, 3) the hydraulic conductivity of the unfrozen soil, 4) the compressibility of unfrozen soil, and 5) the weight of material above the ice lens (15).

Secondary Heave Theory

Usually, the aspect of frost heaving explained by capillary theory is considered as "primary heave". After frequently finding disagreement between the measured and calculated values of heaving pressure using the simple capillary model, Miller (23) concluded that the only kind of ice segregation consistent with the model was the formation of needle ice at the soil surface. In order to supplement the primary frost heave , Miller put forward his "secondary" frost heave theory. He contended that secondary frost heave involves the pore ice under an ice lens, which exists and allows the transport of pore water in a region between ice lens and the freezing front. This region is called the "frozen fringe".

According to Miller (6), the driving force for frost heave in saturated granular soil is the interaction of pore ice, pore water, temperature and the swelling properties of adsorbed films within the frozen fringe.

It is well established that all water in a fine-grained soil does not freeze at a unique temperature and also that under a given set of conditions, some minimum temperature drop below 0^oC is required (i.e. freezing point depression) before pore ice can exist. The finer grained a soil is, the greater the freezing point depression, and the more unfrozen water it will retain under a given set of conditions. This effect is most pronounced for clays for which significant amounts of unfrozen water have been observed at temperatures lower than -10^{o}C, while most of the water in silts freezes within a couple of tenths of a 0 ^{o}C (24).

One relation which is applicable to the ice-liquid mixture in frozen soil is the generalized Clapeyron equation

$$\frac{P_w}{\rho_w} = \frac{P_i}{\rho_i} + \frac{L}{T_o} T \tag{2.3}$$

where:

P_w = pore water pressure.

ρ_w = density of water.

P_i = pore ice pressure.

ρ_i = density of ice.

L = latent heat of fusion

T_o = normal freezing point of water, 273.15(K).

T = absolute current temperature,(K)

Miller also used Eq.(2.1) to describe the relationship between the radius of curvature of the ice/water interface in a pore and the pore ice pressure and pore water tension. Most investigators have been interested in relations of Eq.(2.1) and Eq.(2.3), because they might provide some means of determining the magnitude of suction developed in the pore water in terms of temperature, O'Neill(24).

The most important aspect of the secondary heave model is the impact of regelation. The most familiar demonstration of regelation is mechanically induced (25). It involves a thin wire, with a heavy weight at one end, draped over a block of ice. In this situation the wire will slowly sinks through the solid block of ice. The ice beneath the wire is subjected to high pressure, which, according to Eq.(2.3), lowers the freezing temperature so that the ice melts. The melted water flows up around the wire and promptly refreezes so that the solid block of ice shows no evidence of the passage of the wire through it.

The evidence that an ice lens can form within frozen soil at some distance from the freezing front can not be explained by the capillary theory, but it can be explained by the regelation phenomenon O'Neill and Miller (6). It can be observed that a solid particle embedded in ice moves under conditions of temperature and pressure gradient (26, 27). Koopmans and Miller (28) showed that for non-colloidal soils, suction in the freezing fringe is much higher than the capillary equation can predict. Based on that observation, they noted that there must be some other driving force or some other mode of transport in freezing soils. Koopmans and Miller (28) then postulated that ice moves through the soil by regelation around the soil grains during freezing. Water is more strongly attracted to a solid particle and hence a thin layer of water exists around it (26). Since ice melts at the warmer side or at the lower suction side of the solid particle the resulting water is transported around the particle through the thin liquid layer to the cooler side or to the higher suction side and freezes there (13). This process causes an uneven thickness of the adsorbed film but the surface adsorption forces try to center the grain or solid particle. Consequently, the grain moves towards the warmer or high suction environment inducing further regelation.

In general, according to the secondary heave theory, frost heaving depends on 1) the rate of heat extraction, 2) the size of the soil pores, 3) the freezing point of the water at the base of the growing ice lens, 4) the hydraulic conductivity of the freezing fringe, 5) the temperature gradient within the freezing fringe, 6) the thickness of the freezing fringe, 7) the in-situ moisture tension in the unfrozen soil, 8) the hydraulic conductivity of the unfrozen soil, 9) the compressibility of the unfrozen soil, and 10) the magnitude of the overburden pressure (15).

Adsorption Force Theory

In the previous section, we neglect the adsorbed or film moisture effect, only considering the moisture effect due to capillary theory. Takagi (29, 30) proposed another explanation of frost heaving. He suggested that there is a 'solid-like stress' in the adsorbed film of water between the ice and soil surface, which primarily causes frost heaving. The water film and the soil particle support the weight of the ice lens and the heaving stress is determined by the solid-like stress in the film not the heaving pressure P_i which is derived from Eqs. (2.1)-(2.3) in the other two frost heave theories. Takagi (30) stated that the heaving stress is also limited by the segregation freezing temperature, which cannot be lower than the freezing point of the film water, and that the decisive factor for determining the freezing point depression, and thus the limit of the heaving pressure, is the specific surface area of the soil particles, as suggested by Anderson and Tice (31).

According to Takagi's adsorption force theory, the tension in the pore water is independent of the heaving stress. The origin of the tension is in the film water. The freezing film, in response to the loss of its thickness of the growing ice lens, generates the tension that draws pore water to the region of freezing.

In general, according to segregation freezing theory, frost heaving depends on 1) the rate of heat removal, 2) the freezing point of the film water, 3) the specific surface area of the soil particles, 4) the hydraulic conductivity of the film water, 5) the thickness of the zone of freezing, 6) the temperature gradient in the zone of freezing, 7) the hydraulic conductivity of the unfrozen soil, 8) the compressibility of the unfrozen soil, and 9) the weight of the material supported by the ice lens (15).

Overburden Pressure and Shut-Off Pressure

In Eq.(2.2), there is a total stress σ, which includes the weight of soil above the ice lens and any overburden pressure P. Penner and Ueda (32) and Nixon, J.F. and N.R. Morgenstern (21) show, experimentally that the magnitude of heave decreases as the external overburden pressure is increased on heaving soils. When the overburden pressure is very high, they found that the water from the freezing fringe exudes for a certain time and then reverses its direction of flow. Other researchers have suggested that for any soil there exists an overburden pressure at which the heaving of soil would be prevented completely. This overburden pressure is termed the "shut-off pressure". Konrad and Morgenstern (33) calculated the theoretical values of shut-off pressure from the Clausius-Clapeyron equation assuming the no flow condition, i.e. P_w=0. But Penner and Ueda (32) showed that any overburden pressure just delays the process of water intake and hence there always would be some amount of heave given sufficient time. In practical cases, however, the delay induced by an overburden is beneficial.

Frost Heaving and Ice-Lens Formation Criteria

There are various criteria that frost heave models use to determine when and where ice-lens and heave could occur. For simplicity, some frost heave models monitor the ice content in the freezing fringe and when it exceeds a critical ice content, it is allowed to heave. Taylor and Luthin (34) use a critical ice content equal to 85% of porosity, regardless of unfrozen water content. Others assume that if the ice content is equal to or greater than the porosity minus the unfrozen water, then heave would occur. For some other models, they attribute frost heave to new ice-lens forming, so the issue is when and where the new ice lens will be formed. Konrad and Morgenstern (35) assume that when the existing warmest ice-lens reaches a certain temperature, T_{sm}, a new ice-lens forms where the temperature is T_{sf}. The magnitudes of both T_{sm} and T_{sf} were less than zero and T_{sm} was less than T_{sf}. This criterion means that an ice-lens initiates at temperature T_{sf} and develops till its temperature reaches T_{sm}. O'Neill and Miller (36) assume that when the effective stress at a point in the soil goes to zero (i.e, the neutral pressure in the soil surpasses the overburden pressure) a new ice-lens will form. They use the stress partition factor to determine the neutral pressure. That is

$$\sigma_n = \chi P_w + (1-\chi) P_i \tag{2.4}$$

where:

σ_n is the neutral pressure which equals σ-σ_s

χ is a stress partition factor

This last factor can be evaluated in several ways. It mainly depends on the ice content, unfrozen water content and porosity of soil. There are experimental results and empirical formulas available for prediction of the factor. As the ice content increases, we expect the contribution of ice pressure to the neutral stress to increase, which means that the stress partition factor must decrease. O'Neill and Miller (36) used the following expression for χ based on experimental observations:

$$\chi = \left(\frac{W}{n}\right)^{1.5} \tag{2.5}$$

where:

n is porosity of soil.

W is volumetric unfrozen water content.

CHAPTER 3

GOVERNING EQUATIONS FOR FROST HEAVE

Assumptions

Frost heave is a very complicated phenomenon and numerical modeling is impossible without making a number of reasonable assumptions. Existing frost heave models usually assume that:

- the soil is saturated and isotropic,
- the soil is solute and ion free,
- the soil grains are incompressible,
- the pore water pressure and water flow are related by Darcy's law.
- there is no mass transport beyond the warmest ice-lens, and
- the freeze-thaw cycle has no effect on the frozen soil properties.

Definition of Terms

When modeling frost heave, it is necessary to solve several transport equations (e.g. mass and energy conservation). These conservation equations should be applied at a microscopic level where we focus our attention on what happens at a point within a given phase (water, ice or soil). These microscopic equations cannot be easily solved, because the geometry of the surface that bounds each individual phase is complicated in nature and may not be observable (37). Hence, the description and solution of a transport problem at the microscopic level is impractical. Therefore, another level of description is needed, namely the macroscopic level, at which measurable, continuous and differentiable quantities may be determined and boundary-value problems can be stated and solved (37). Following Bear (37) and Ni and Beckermann (38), the macroscopic governing equations can be derived by averaging the microscopic equations over a finite sized averaging volume that contains a number of phases (e.g. solid, liquid, etc). This volume, called a Representative Elementary Volume (REV), is much smaller than the system but large compared to the characteristic size of the interfacial structures. Assuming the volume of the REV is V_o, each phase k in V_o occupies a volume V_k and is bounded by the interfacial area A_k. The symbol n_k is the outwardly directed unit normal vector on the interface A_k, and w_k is the velocity of the interface A_k.

For any quantity, say 'E', the definition of the volume average of E in phase k is :

$$< E_k > = \frac{1}{V_o} \int_{V_o} X_k E_k \, dV \tag{3.1}$$

where:

X_k is a phase function, being equal to unity in the phase k and zero outside the phase k.

E_k is the value E at a microscopic point in phase k.

The intrinsic volume average of E is defined as

$$< E_k >^k = \frac{1}{V_k} \int_{V_o} X_k E_k \, dV \tag{3.2}$$

where:

V_k is the volume of phase k within volume V_o.

The volumetric fraction of phase k is defined as

$$\theta_k = \frac{V_k}{V_o} \tag{3.3}$$

such that

$$< E_k > = \theta_k < E_k >^k \tag{3.4}$$

When deriving macroscopic transport equations, rules for averaging of a time derivative and a spatial derivative are needed, they are (37):

$$< \frac{\partial E_k}{\partial t} > = \frac{\partial < E_k >}{\partial t} - \frac{1}{V_o} \int_{A_k} E_k \, w_k \cdot n_k \, dA \tag{3.5}$$

$$< \nabla E_k > = \theta_k \nabla < E_k >^k + \frac{1}{V_o} \int_{A_k} E_k \, n_k \, dA \tag{3.6}$$

Microscopic Transport Equations in Frost Heave

Since frost heave problems involve displacements caused by heat transfer in porous media, we need to use the mass, momentum and energy transport equations to model the phenomena. To the author's knowledge, most of the frost heave models used so far only consider the coupled mass and heat transport equations and not the momentum equation. The mass and energy equations at a microscopic point are as follows:

Mass:

$$\frac{\partial}{\partial t}\rho_k + \nabla(\rho_k v_k) = 0 \tag{3.7}$$

where:

ρ_k =density of phase k at a microscopic point.

v_k =velocity of phase k at a microscopic point.

Energy:

$$\frac{\partial}{\partial t}(\rho_k H_k) + \nabla(\rho_k H_k v_k) = -\nabla q_k \tag{3.8}$$

where:

H_k =specific enthalpy of phase k at a microscopic point.

q_k =heat flux in phase k.

Macroscopic Transport Equations in Frost Heave

Mass Conservation Equation:

On averaging the microscopic mass equation Eq.(3.7) over the REV and employing the appropriate volume averaging rules Eq.(3.5) and Eq.(3.6), the macroscopic mass conservation for the phase k can be obtained (38).

$$\frac{\partial}{\partial t}(\theta_k \rho_k) + \nabla \cdot (\theta_k \rho_k < v_k >^k) = \Gamma_k \tag{3.9}$$

where:

θ_k is volumetric fraction of the k phase

$<v_k>^k$ is intrinsic velocity of the k phase

Γ_k is interfacial transfer term due to phase change

In a frost heaving problem, there are three phases present, i.e. soil, water and ice. Therefore three macroscopic mass conservation equations in the form Eq.(3.9) are required. On adding the equations together, the interfacial transfer term Γ_k will be canceled out. If, further, we neglect the velocity of soil particles and ice in comparison with that of water, and use Darcy's law, we will arrive at the following mixture mass conservation equation,

$$\rho_s \frac{\partial \theta_s}{\partial t} + \rho_w \frac{\partial \theta_w}{\partial t} + \rho_i \frac{\partial \theta_i}{\partial t} - \rho_w \nabla \cdot (K \nabla h) = 0 \tag{3.10}$$

where:

ρ_s, ρ_w, and ρ_i are density of soil particle, water and ice respectively,(assumed to be constant in our consideration).

K is hydraulic conductivity

h is pressure head

θs, θw, and θi are volumetric fraction of soil particles, unfrozen water and ice respectively, i.e.

$$\theta_w + \theta_i + \theta_s = 1 \tag{3.11}$$

In one dimensional case, Eq (3.10) can be written as

$$\rho_s \frac{\partial \theta_s}{\partial t} + \rho_w \frac{\partial \theta_w}{\partial t} + \rho_i \frac{\partial \theta_i}{\partial t} - \rho_w \frac{\partial}{\partial x}(K \frac{\partial h}{\partial x}) = 0 \tag{3.12}$$

Energy Conservation Equation:

Similar to the mass conservation equation, the macroscopic energy conservation equation for each phase can be written as

$$\frac{\partial}{\partial t} \rho_k \theta_k < H_k >^k + \nabla \cdot (\rho_k \theta_k < H_k >^k < v_k >^k) = -\nabla \cdot (< q_k > + < q_k >^t) + Q_k \tag{3.13}$$

where:

$<q_k>^t$ is the dispersive heat flux

Q_k is the net heat taken in from other phases

w_k is the interfacial velocity

According to Bear (37), we can make an assumption that there is thermal equilibrium between the phases, i.e., the averaged temperature T in the REV is the same for all the phases. Further, for computation convenience, if we assume all the material and thermal properties (i.e. density, heat capacity and thermal conductivity) are constant in each phase and velocity of each phase are constant within the REV, and use the Fourier Law of heat conduction, then Eq.(3.13) becomes,

$$\frac{\partial}{\partial t}\rho_k \theta_k < H_k >^k + \nabla \cdot (\theta_k \rho_k < H_k >^k v_k^k) = \nabla \cdot (\theta_k K_{hk} \nabla T) + Q_k' \tag{3.14}$$

where:

K_{hk} is heat conductivity of phase k.

Q'_k is the net heat taken in from other phases

Eq.(3.14) is applicable to each of the phases present, e.g. soil, water and ice. After adding the three equations together, and eliminating the interfacial term Q'_k, the 'mixture' energy equation is:

$$\frac{\partial}{\partial t}\overline{H} + \nabla \cdot v\overline{H} = \nabla \cdot (\overline{K}_h \nabla T) \tag{3.15}$$

where:

$$\overline{H} = \theta_s \rho_s < H_s >^s + \theta_w \rho_w < H_w >^w + \theta_i \rho_i < H_i >^i$$
$$v\overline{H} = \theta_s \rho_s v_s < H_s >^s + \theta_w \rho_w v_w < H_w >^w + \theta_i \rho_i v_i < H_i >^i$$
$$\overline{K}h = \theta_s k_s + \theta_w k_w + \theta_i k_i$$

k_s, k_w, k_i = thermal conductivity of soil particles, unfrozen water and ice respectively.

In one dimension, Eq (3.15) can be written as

$$\frac{\partial}{\partial t}\overline{H} + \frac{\partial}{\partial x}(\overline{vH}) = \frac{\partial}{\partial x}(\overline{K}_h \frac{\partial T}{\partial x}) \tag{3.16}$$

Simplifying the Energy Equation:

By definition of enthalpy (8) and an assumption of constant material properties over time and space, the enthalpy of soil, water and ice can be respectively written as:

$\langle H_s \rangle^s = c_s T$,

$\langle H_w \rangle^w = c_w T + L$,

$\langle H_i \rangle^i = c_i T$,

where:

c_s, c_w, c_i = specific heat capacity of soil particles, unfrozen water and ice respectively.

L = latent heat of water.

In this case, Eq.(3.16) can be written in terms of temperature as,

$$\frac{\partial(\overline{C}\,T)}{\partial t} + \frac{\partial}{\partial x}(\overline{vH}) = \frac{\partial}{\partial x}\left(\overline{K}_h \frac{\partial T}{\partial x}\right) - \rho_w L \frac{\partial \theta_w}{\partial t} \tag{3.17}$$

where:

$$\overline{C} = \theta_s \rho_s c_s + \theta_w \rho_w c_w + \theta_i \rho_i c_i$$

is a mixture heat capacity.

Throughout the remainder of the work, Eq.(3.12) and Eq.(3.17) will be the governing equations used.

CHAPTER 4

MODELLING OF FROST HEAVE

Though the frost heave process is not fully understood existing frost heave models can provide worthwhile qualitative estimates of heaving. The models discussed in Chapter 2 focus on micro-scale process and attempt to predict quantities such as the growth of ice lenses, e.g. the 'Rigid Ice Model', O'Neill and Miller (6). When trying to predict the response of field scale problems, however, it is better to develop a lumped parameter model which considers the global effects (i.e. the response of the whole mixture). Examples of a micromechanical micro-scale model and a lumped parameter model follow.

The Rigid Ice Model (RIM)

This model focuses on the micro-process of ice lens formation. In this model the porosity is treated as constant, the amount of heave is due to the ice lens movement V_I. To date, this model has only been applied to one-dimensional cases.

The basic assumption in the RIM is that the pore ice in the freezing fringe and the warmest ice-lens are inter-connected and form an intricate and rigid body of ice. Although this 'rigid ice' assumption has not been experimentally verified, O'Neill and Miller (6) provided some rational explanations to support it.

Since there are more than two unknowns in Eq.(3.12) and Eq.(3.17), an additional relation is needed to solve them. O'Neil and Miller (6) propose that the unfrozen water fraction θ_w is a function of temperature T and the difference of ice pressure P_i and water pressure P_w defined by ϕ_{iw}. Note that this relation can be obtained from experiments. The pressure difference, ϕ_{iw}, may be expressed as a linear function of water pressure and temperature by using the Clausius-Clapeyron Eq.(2.3). That is

$$\phi_{iw} = (\frac{\rho_i}{\rho_w} - 1)\, P_w - \frac{\rho_i L}{273.0} T = AP_w + BT \qquad (4.1)$$

where: A and B are material constants. From Eq.(4.1), we can write

$$d\theta_i = \frac{\partial \theta_i}{\partial P_w} dP_w + \frac{\partial \theta_i}{\partial T} dT = AI' dP_w + BI' dT \tag{4.2}$$

where $I' = d\theta_i / d\phi_{iw}$.

O'Neil and Miller (6) propose that the ice movement will determine the frost heave. From the rigid-ice assumption, the ice flux v_i can be written as

$$v_i = V_I \theta_i \tag{4.3}$$

where:

V_I = ice lens velocity.

By using Eq.(4.2) and Eq.(4.3), the mass governing equation Eq.(3.12) can be written as

$$(\Delta\rho AI') \frac{\partial P_w}{\partial t} + (\Delta\rho BI') \frac{\partial T}{\partial t} - \frac{\partial}{\partial x}[\frac{k}{g}(\frac{\partial P_w}{\partial x})] + \rho_i V_I [AI' \frac{\partial P_w}{\partial x} + BI' \frac{\partial T}{\partial x}] = 0 \tag{4.4}$$

where

$\Delta\rho = \rho_i - \rho_w$

g =gravitational acceleration.

Similarly, on neglecting the convection term $\frac{\partial}{\partial x}(\overline{vH})$, the energy governing Eq.(3.17) can be written as

$$(\overline{C} - \rho_i LBI') \frac{\partial T}{\partial t} - \rho_i LAI' \frac{\partial P_w}{\partial t} - \rho_i LV_I [AI' \frac{\partial P_w}{\partial x} + BI' \frac{\partial T}{\partial x}] - \frac{\partial}{\partial x}(\overline{K_h} \frac{\partial T}{\partial x}) = 0 \tag{4.5}$$

Both the above equations contain the same three unknowns, namely water pressure, P_w, temperature, T and ice velocity, V_I. Hence an additional equation is required to determine the ice velocity V_I.

O'Neil and Miller (6) develop a formula using the mass balance over the entire length of whole (lens free) soil to calculate V_I. The total mass flux through the plane at x_b is given by $\rho_i V_I$ while the mass flux at the warm end of the column is given by $(\rho v)_w$, as evaluated at x_w using Darcy's law. The difference between these two fluxes must equal the rate of change of the aggregate mass content in between,

$$\rho_i V_I - \rho_w v_{(x_w)} = \frac{d}{dt}\int_{x_b}^{x_w}[\rho_w \theta_w + \rho_i \theta_i]\,dx \tag{4.6}$$

or

$$V_I = \frac{\rho_w}{\rho_i} v_{(x_w)} + \frac{\Delta\rho}{\rho_i}\frac{d}{dt}\int_{x_b}^{x_w}\theta_i\,dx \tag{4.7}$$

After specifying suitable initial and boundary conditions, Eq.(4.4), Eq.(4.5) and Eq.(4.7) represent a closed set in the unknowns, namely water pressure, temperature and ice velocity. After getting the ice lens velocity V_I, the heave can be calculated as follows,

$$S(t) = \int_o^t V_I\,dt \tag{4.8}$$

Since the formation of ice lenses is not continuous, in order to calculate the new ice lens velocity, we must know when and where a new lens will be initiated. The various available criteria to determine when and where a new ice-lens forms were discussed in Chapter 2. O'Neill and Miller (6) used the concept of the stress partition factor to determine the location and the time for the formation of a new ice-lens. This criterion predicts the formation of new ice-lens somewhere behind the freezing front rather than at the freezing front. This is in agreement with experimental evidence (39). An ice-lens is a macroscopic discontinuity in the soil fabric and hence it can form only by moving the soil grains apart. This would be possible when grain-to-grain contact stresses, i.e. the effective stress, becomes zero.

The Lumped 'Macro-Scale' Frost Heave Model

Though the Rigid Ice Model is popular and well-verified by experiment, the current stage of development of the model only allows one-dimensional freezing simulation. This is because this microscopic approach is limited to the investigation of small-scale problems in simplified domains and it is extremely difficult to measure ice velocity at each location.

To avoid the complexity of modeling micro-scale frost heave, the alternative is to use a so called "lumped parameter" frost heave model. This model, first presented by Blanchard and Fremond (1), is a macro-scale model which captures the micromechanical process of ice lens formation and the associated frost heave via a constitutive relationship for the porosity rate. In effect, the ice growth is described as the average increase in porosity (volume) of a soil element rather than separate ice lens growth as in the RIM. Michalowski and Voller (40) use the analogy of mechanics of solids to explain the idea of this lumped frost heave model. From theories of the mechanics of solids, it has been shown that phenomenological models based on introducing material parameters on the macroscopic level can be more successful in predicting such global quantities as displacements and limit loads than using micromechanics-based models. One of the simplest examples of a phenomenological model is Hook's model of linear elasticity. Unlike the use of a micromechanical-based model, it cannot explain why the deformation is elastic, but it can be quite accurate in determining the magnitudes of deformation or the stress state when material parameters such as Young's modulus and Poisson's ratio are given. These parameters are taken from macroscopic level experiments.

The specific assumptions used in the lumped macro-scale model are the following:

- the frost heave accumulation in the frozen soil can be described by a constitutive equation: i.e, the porosity rate is a known function of freezing temperature (see Fig 4.1),
- the unfrozen water content of the frozen soil depends on temperature,
- the frozen soil is a viscoelastic material for which mechanical properties depend on temperature.

In this model, the key is the porosity rate function. The porosity rate function must account for characteristic features of the freezing process of frost susceptible soil known from laboratory tests. In Frémond's model, it is assumed that rate of porosity $\dot{n}$ is only a function of temperature, which is expressed as

$$\dot{n} = \dot{n}_m \frac{T}{T_m} e^{1-\frac{T}{T_m}} \tag{4.9}$$

where:

$\dot{n}_m$: maximum rate of porosity increase

T_m: the temperature where $\dot{n}_m$ occurs

Eq.(4.9) can be written in a general form

$$\dot{n} = -\alpha T e^{(\beta T)} \tag{4.10}$$

where, α and β are quantities to be determined from experimental data. In addition to the rate of porosity change function, the relation of unfrozen water content θ_w and temperature T is also needed, which can be obtained from experiments.

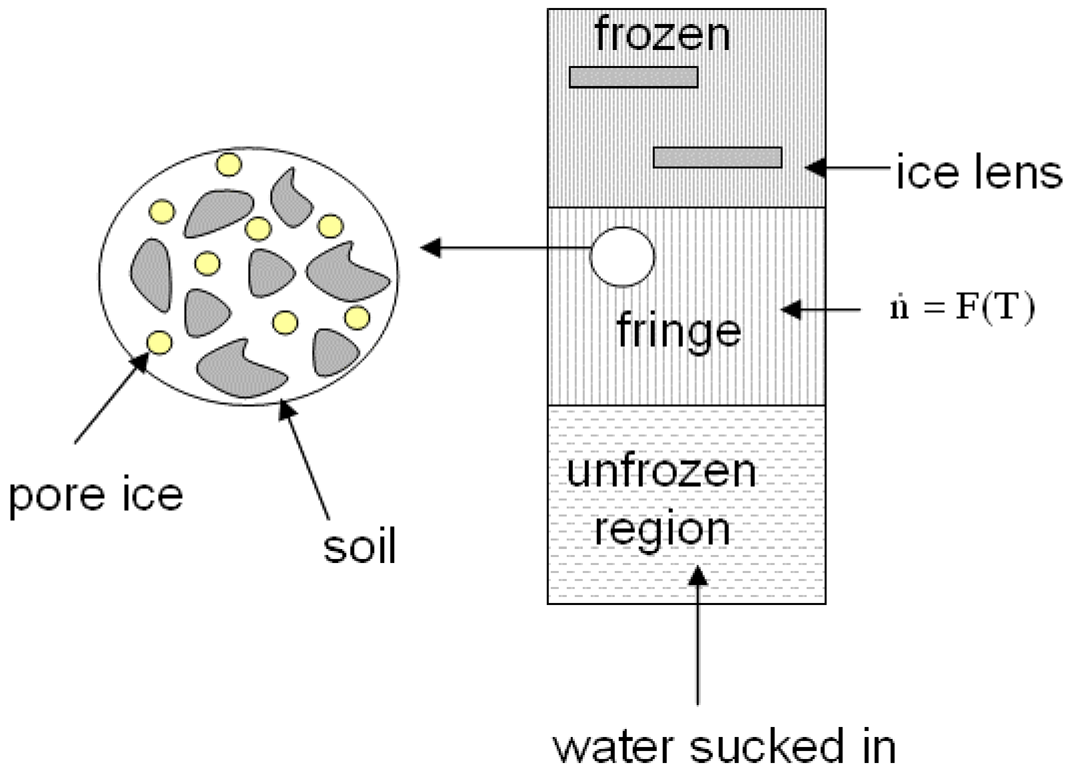


Figure 4.1 Schematic of Porosity Rate Model

Using the porosity rate and water content sub-models, we can solve the mass and energy governing equations Eq.(3.12) and Eq.(3.17) to get temperature, moisture content and pore pressure fields.

An effort was undertaken at Laboratory Central des Ponts et Chausses (Paris) to evaluate parameters α and β from experiments. The value of β for frost-susceptible soils appears to be of the order of unity, while the value of α is of the order of 10^{-6} to 10^{-8} (41). The values of α and β describe the evolution of the porosity within the soil. To avoid costly experimentation these are derived from the average temperature of whole domain and the average heave rate for each time (42). This can lead to the situation where the temperature distribution from the model turns out to be reasonable, but the pore pressure field and stress-strain field from the model can be unrealistic (43). Nevertheless, the idea of using a porosity rate function has great potential. The major advantage of this lumped model is that it is easy to extend to a two-dimensional model without any consideration of the direction of ice lens movement.

CHAPTER 5

THE LUMPED MACRO-SCALE MODEL: MODEL SPECIFICATION

The success of the lumped macro-scale model depends on how accurately the factors that affect the porosity growth are captured. As was noted in the previous chapter, there is some uncertainty in the porosity rate function, and an investigation of the porosity rate function sub-model is merited. In a modification of Fremond's model, Michalowski and Voller (40) considered porosity growth as a function of temperature, temperature gradient, current porosity and stress condition. In this approach porosity growth depends on five parameters.

The most useful constitutive model usually is one that can simulate the behavior of the material to an acceptable degree of accuracy with the least number of parameters introduced. Ideally, these parameters should be based on experimental measurements. But, due to the difficulty of developing experimental techniques to measure each parameter, this is not always possible. The value of the parameter can be optimized, however, using parametric computer simulations to find the parameter values which match experimental results as closely as possible.

The objective of this study is to investigate the lumped macro-scale frost heave model without considering overburden pressure and pore water pressure effect on the porosity rate sub-model. This will be carried out by consideration of frost heave in a one-dimensional domain. In order to set up the model, we need to provide: 1) a governing equation, 2) a porosity rate function, 3) an unfrozen water content vs. temperature relationship and 4) a heave simulation model. Focus will be placed on the parameters used to describe the porosity rate and the water content sub-models.

Domain and Governing Equation

In a one-dimensional situation, frost heave can be determined from the solution of the heat equation Eq.(3.17), i.e.,

$$\frac{\partial(\bar{C}T)}{\partial t} + \frac{\partial}{\partial x}(\overline{vH}) = \frac{\partial}{\partial x}(\bar{K}_h \frac{\partial T}{\partial x}) - \rho_w L \frac{\partial \theta_w}{\partial t} \qquad (5.1)$$

coupled with sub-models (e.g. porosity rate function $\dot{n}$), initial and boundary conditions which are illustrated in Fig 5.1.

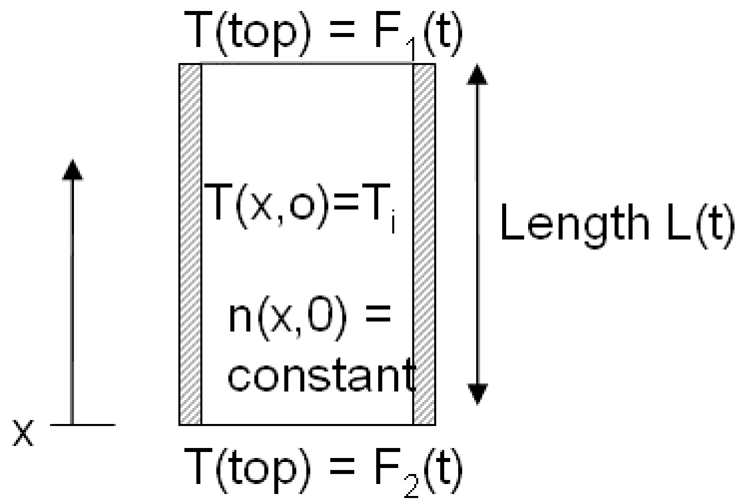


Figure 5.1 The Solution Domain

Appropriate forms of the sub-models are discussed below

Porosity Rate Function

The porosity rate model adopted for this research is based on the model suggested by Michalowski and Voller (40) which is an extension of Fremond's model. This is,

$$\dot{n} = \dot{n}_m \frac{T}{T_m} e^{1-\frac{T}{T_m}} (1 - e^{\varepsilon\frac{\partial T}{\partial x}})(1 - n)^{\gamma} \quad (5.2)$$

Note that there are four unknown parameters $\dot{n}_m$, T_m, ε and γ in this sub-model. Where ε is the "temperature gradient factor" in the porosity rate sub-model $\dot{n}$, and γ is the "exponential factor of porosity" in the porosity rate sub-model $\dot{n}$.

The function, Eq.(5.2), is constructed in such a way that its first part, $\dot{n}_m \frac{T}{T_m} e^{(1-\frac{T}{T_m})}$, describes the porosity rate as Frémond's model did, while the next two terms are dimensionless factors, all ranging from 0 to 1. The first term expresses the dependence of the porosity rate on temperature and is controlled by the maximum rate of porosity increase, $\dot{n}_m$ and the temperature where $\dot{n}_m$ occurs, T_m. With this term, as the temperature drops below 0 ^{o}C the porosity rate increases from zero to a maximum, and then diminishes back to zero for temperatures well below 0 ^{o}C. This behavior matches real freezing soil situations where, due to the freezing depression, water can exist at temperatures below 0°C. "The rapid increase of porosity at temperatures slightly below 0 ^{o}C can be treated as primary frost heave, while its slow growth at lower temperature can be viewed as the secondary frost heave process" (40).

The second component of the porosity rate function $(1-e^{\varepsilon\frac{\partial T}{\partial x}})$ expresses the dependence of the porosity rate on temperature gradient. The coordinate x is from the warm side of the freezing front toward the cold side, hence, $\partial T/\partial x<0$. We can see that, when $|\partial T/\partial x|$ is large, this term is close to unity, and when $|\partial T/\partial x|$ is small, this term drops to zero. This behavior of this term can be explained in the following way (40): let's assume that temperature is the only factor influencing cryogenic suction, so the suction gradient must be zero when $\partial T/\partial x=0$. According to Darcy's law, no water influx takes place at this time so that no heave takes place. If there is temperature gradient applied through the soil, due to the cryogenic suction, the additional water will be sucked towards the freezing front to cause frost heave. According to experimental observation, however, if the temperature gradient is too large, then soil freezing will be very fast, there is not enough time to let additional water enter the frozen soil so that no heave takes place.

The third component of the porosity rate function, $(1-n)^{\gamma}$, expresses the dependence of the porosity rate on the current porosity. This term will cap the porosity at a maximum value of 1.

Unfrozen Water Content

The relationship of unfrozen water content ω in terms of the temperature T is discussed in many prior papers. Based on experimental tests (44, and 45), the unfrozen water content can be assumed as,

$$\omega = \omega_r + (\overline{\omega} - \omega_r)e^{\eta T} \qquad (5.3)$$

where:

ω is the mass fraction of unfrozen water content.

ω_r is the residual unfrozen water content at a very low temperature.

$\overline{\omega}$ is the minimum unfrozen water content at 0 ^{o}C

η is the "exponential factor of temperature" in the water content sub-model ω.

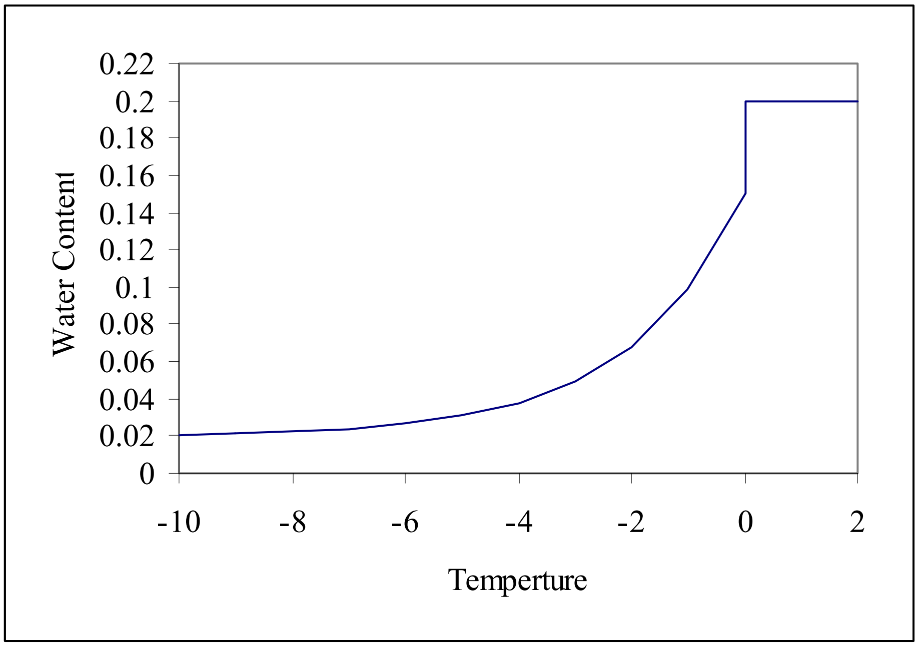


Figure 5.2 Water Content vs. Temperature

Fig 5.2 shows a typical water content and temperature relationship. Note that 1) the step discontinuity at T=0°C indicates the sudden mass fraction change of water due to the nucleation when releasing the latent heat and 2) the "tail" indicates the unfrozen water existence below 0°C due to the depression of the freezing point.

In the governing equation Eq.(5.1), we use volumetric fraction instead of mass fraction, so the relationship needs to be converted to volumetric terms. This is done using the basic soil phase relationship (46),

$$\theta_w = \omega \frac{\rho_s}{\rho_w}(1 - n) \tag{5.4}$$

Heave Calculation

By utilizing to the consolidation model in soil mechanics (46), we can obtain the following expression for heave,

$$\Delta S = S_o \frac{\Delta e}{1 + e_o} \tag{5.5}$$

where:

ΔS: change in sample height

S_o: initial height of sample

Δe: change of void ratio

e_0: initial void ratio

To convert to an equation involving the porosity, n, instead of void ratio, e, the following relationships

$$e = \frac{n}{1 - n}$$

$$\Delta e = e - e_0 = \frac{n - n_0}{(1 - n)(1 - n_0)}$$

are used to get

$$\Delta S = S_o \frac{n - n_0}{1 - n} \tag{5.6}$$

Eq.(5.6) assumes a constant porosity throughout the sample. In general, this is not the case and the heave needs to be calculated as

$$\Delta S = \int_{x=0}^{x=S_o} \frac{n - n_0}{1 - n} dx \tag{5.7}$$

CHAPTER 6

NUMERICAL SOLUTION AND VERIFICATION

Numerical Methodology

Calculation of the frost heave from the proposed model requires the numerical solution of Eq.(5.1) with appropriate coupling to the porosity rate sub-model Eq.(5.2) and the unfrozen water content sub-model Eq.(5.3). This problem falls into the class of phase change problems, which requires the tracking of a moving front on which heat balance conditions have to be met. The classical approach to deal with moving boundary problem is the Stefan approach. Numerical solutions based on this approach require the accurate tracking of the phase change boundary, which, in turn, requires that the numerical space grid deforms. A popular alternative approach is the so-called enthalpy method. The major advantage of this method is that despite the presence of moving boundaries, the problem is cast in a conservative form that allows for the solution to be obtained on a fixed space grid. There are a variety of ways in which the enthalpy methods can be implemented (47). Two common ways are:(1) the apparent heat capacity methods (48, 49), in which the nonlinearity associated with the evolution of latent heat is accounted for using a modified heat capacity term, and (2) the source based methods (50, 51, and 52), in which the latent heat evolution is represented by a suitable source term. In this work, the second method, i.e. source based method, will be applied to the solution of frost heave.

Using a fully-implicit control-volume finite difference scheme, Eq.(5.1) can be discretized as (see Patankar 1980 (53) and Fig 6.1)

$$a_P T_P = a_N T_N + a_S T_S + B_P \tag{6.1}$$

where:

$$a_N = K_N(dt/dx^n)$$
$$a_S = K_S(dt/dx^s)$$
$$a_P = \overline{C}dx + a_N + a_S$$
$$B_P = \overline{C}^{old}dxT^{old} + \rho_w Ldx(\theta_P^{old} - \theta_P) + \text{'convec.term'}$$

θ_P is unfrozen water content at node P the 'convecection term' is a discretized form of $\frac{\partial}{\partial x}(\overline{vH})$.

Note that according to Fig 6.1, this scheme has been applied on a continual deforming grid of control volumes.

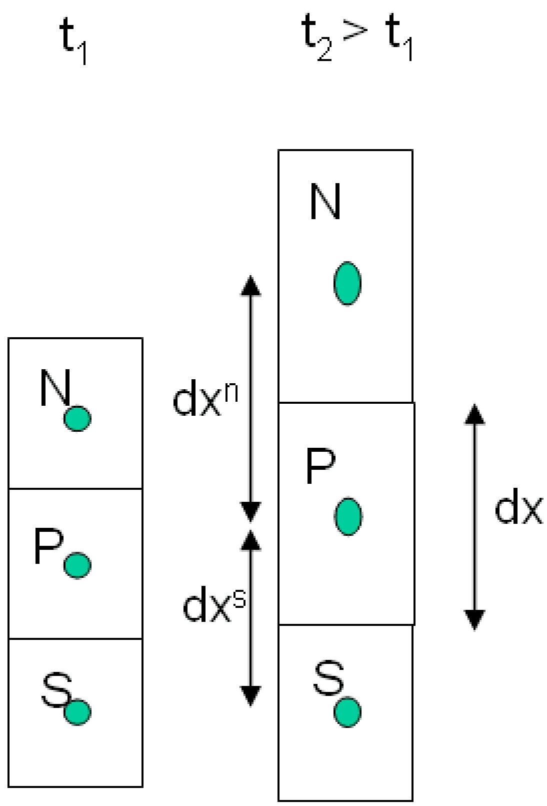


Figure 6.1 Arrangement of Control Volumes

Eq.(6.1) is non-linear and an iteration procedure is needed to get consistent results between temperature, unfrozen water content and porosity. In arriving at an iterative solution, Eq. (6.1) is written as

$$a_P T_P^{m+1} = a_N T_N^{m+1} + a_S T_S^{m+1} + B_P \qquad (6.2)$$

where m is the iteration level

The basic steps in the iteration are

1) guess $(\theta_p)^m$

2) solve Eq (6.2) to get T^{m+1}

3) update water content $(\theta_p)^{m+1}$ from the sub-model Eq.(5.3).

Precise details are given below.

Representation of the Convection Term:

In arriving at a representative of the convection term, the basic assumption that the ice and soil velocity are the same is made, i.e, V_s=V_i=V. Further on the deforming grid in Fig 6.1, the control volume interface velocity is set to V. In this case, a given volume of soil and ice will always remain in the same control volume and only water will cross the boundary of the control volume. So the only contribution to make the porosity increase in a volume is due to the water movement through that volume, and in this case, the 'convec.term' in Eq(6.2) can be calculated as

$$\text{convec.term} = \theta_P^m \rho_w (c_W T_P^m + L)(dx^m - dx^{old}) \tag{6.3}$$

Linearization:

To fashion a solution Eq(6.2) needs to be linearized. As we know, the unfrozen water content has a step discontinuous at T=0°C, which will make the problem strongly non-linear. Here the so called "general slope method" (52) is used to linearize Eq.(6.2). Using a Newton expansion, we can write

$$\theta_P^{m+1} = \theta_P^m + \left[d\theta_P / dT\right]^m (T^{m+1} - T^m) \tag{6.4}$$

and a linearized form of Eq(6.2) follows, i.e,

$$(a_P - S_P) T_P{}^{m+1} = a_N T_N{}^{m+1} + a_S T_S{}^{m+1} + S_c \tag{6.5}$$

where

$$S_P = -\rho_w L dx^m \frac{d\Theta_P}{dT}$$

$$S_c = \overline{C}^{old} dx^{old} T^{old} + \rho_w L dx^m [\theta_P^{old} - \theta_P^m + \frac{d\theta}{dT} T^m] + \theta_P^m \rho_w L(c_W T^m + L) + (dx^m - dx^{old})$$

Algorithm:

The following algorithm is used to solve Eq.(6.5):

1. Set time step dt, initial space step dx and all material properties
2. Set initial and boundary condition including temperature, water content, porosity, etc.
3. Begin time loop
4. Begin iteration loop

   - calculate all coefficients of Eq.(6.5) using previous iterate values where appropriate. Set the 'slope' $d\theta_p/dT$ to an appropriate value as follows:

$$\frac{d\theta_P}{dt} = \begin{cases} \dfrac{c_i}{L} & \text{if } \theta_P \geq n_P \\ 10^{10} & \text{if } \overline{\theta} < \theta_P < n_P \\ (\overline{\omega} - \omega_r)\eta e^{\eta T} \dfrac{\rho_s}{\rho_w}(1-n) & \text{if } \theta_r < \theta_P < \overline{\theta} \end{cases} \qquad (6.6)$$

   where

   n_p is the current porosity at node p.

   θ_r is the unfrozen volumetric water content when temperature is far below 0°.

   θ is the minimum unfrozen volumetric water content at 0°.

- solve the system equations represented by Eq.(6.3).
- calculate temperature gradient at each node.
- calculate new porosity at each node according to the porosity rate from Eq(5.2).

  $n^{m+1} = n^{old} + \dot{n}dt$

- update water content θ_p from Eq.(6.4):

$$\theta_p^{m+1} = \theta_p^{m} + \left(\frac{d\theta_p}{dT}\right)^{m} (T_p^{m+1} - T_p^{m}) \qquad (6.7)$$

Apply Eq.(6.7) at every node and then use the over/under shoot correction to account for the non-phase change nodes.

$$\theta_p = \begin{cases} 0 & \text{if} \quad \theta_p \le 0 \\ n_p & \text{if} \quad \theta_p \ge n_p \end{cases} \tag{6.8}$$

- after calculating of $(\theta_p)^{m+1}$ the nodal temperature at phase change node should be updated to be consistent with unfrozen water content, i.e.

$$T_p{}^{m+1} = \frac{\mathrm{Log}\left[\frac{(\omega_p - \omega_r)}{(\bar{\omega} - \omega_r)}\right]}{\eta} \tag{6.9}$$

- calculate each new control volume dx^{m+1} based on porosity increase.
- repeat all steps in 4) until convergence is achieved, i.e. the residual of Eq.(6.5) less than 10^{-6}.

5. After convergence, calculate the heave from Eq.(5.7).
6. Move to the next time and repeat until maximum time of analysis.

The "general slope method" is very efficient for this problem, the average iteration number in each time step is only two or three times.

Verification of the Algorithm

Hou (54) considers a simple one-dimensional freezing phase change problem is considered to verify the numerical algorithm that the phase change part of the algorithm works. Complete details of this verification can be found in Hou (54).

A Test Problem

The test problem which is used to calibrate the model is taken from Konrad and Morgenstern (36). The test configuration consists of a cylindrical soil sample 0.18 m in height. The sides of the sample are perfectly insulated so that heat flow is only in the vertical direction. The soil is Denvon silt which is very frost-susceptible and the soil in the experiment is fully saturated. Initially the soil sample is at the uniform temperature of T=1°C. At time t=0, the temperature of the top surface (x= 0.18 m) is lowered and fixed at the temperature of (T=-6°C). The bottom of sample is exposed to an additional water source so that the water can be sucked into the soil to cause heave. During the experiment, the length of the sample increases. The value (i.e, new length - original length) is defined as the "heave" (see Fig 6.2). The experimental result in terms of the heave history is shown as the solid line in Fig 6.3.

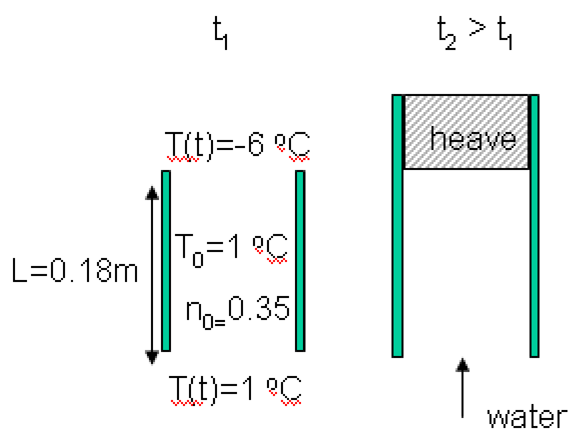


Figure 6.2 The Konrad and Morgenstern Test Problem

Thermal and Material Properties

The thermal and material properties appropriate for the experiment, chosen from Williams and Smith (13), are given in Table 6.1.

Table 6.1 Values of Constant Thermal & Material Properties for the Simulations

soil particle density, ρ_s	2650 (Kg/m^2)
soil particle heat capacity, c_s	900 (J/Kg°C)
soil particle conductivity, k_s	2.92 (W/m°C)
water density, ρ_w	1000 (Kg/m^2)
water heat capacity, c_w	4180 (J/Kg°C)
water conductivity, k_w	0.56 (W/m°C)
ice density, ρ_i	917 (Kg/m^2)
ice heat capacity, c_i	2100 (J/Kg°C)
ice conductivity, k_i	2.24 (W/m°C)
water latent, L	3.3×10^5 (J/Kg)
initial porosity, n_o	0.35

In addition to these property values, values are also needed to specify the parameters in the porosity rate and water content sub-models Eq.(5.2) and Eq.(5.3), i.e,

$\dot{n}_m$ is the maximum rate of porosity increase

T_m is the temperature where $\dot{n}_m$ occurs

ε is the "temperature gradient factor" in the porosity rate sub-model $\dot{n}$.

γ is the "exponential factor of porosity" in the porosity rate sub-model $\dot{n}$.

ω_r is the residual unfrozen water content at a very low temperature

$\overline{\omega}$ is the minimum unfrozen water content at 0 ^{0}C.

η is the "exponential factor of temperature" in the water content sub-model ω.

For these seven parameters, some of them can be directly obtained from experimental results. The maximum porosity rate $\dot{n}_m$ can be obtained on calculation of the steepest slope of the heave result in Fig 6.3, the value of $\dot{n}_m$ should be of the order of 10^{-5} to 10^{-4}. Based on the temperature depression in frozen soil, T_m should have a value slightly below 0°C (~ -0.1 ^{0}C). The three parameters (ω_r, $\overline{\omega}$ and η) that define the water content sub-model can be obtained from water content experiment results. The remaining two parameters (i.e, ε and γ) are adjusted in order to give a best fit to the experimental heave results. The best-fit values referred to a "best values" are given in Table 6.2. The comparison of the predicted heave and the experiment result using 30 numerical grid and 120 seconds time step is shown in Fig 6.3.

The numerical results match the experiment quite closely. The major discrepancy occurs at a later time. This is due to the fact that the last term $(1-n)^\gamma$ in the porosity rate sub-model Eq(5.2) can not reach zero.

Table 6.2 Base Property Values

$\dot{n}_m$	10^{-4}
T_m	-0.2°C
ω_r	2%
$\overline{\omega}$	15%
η	0.5
ε	0.5
γ	6.5

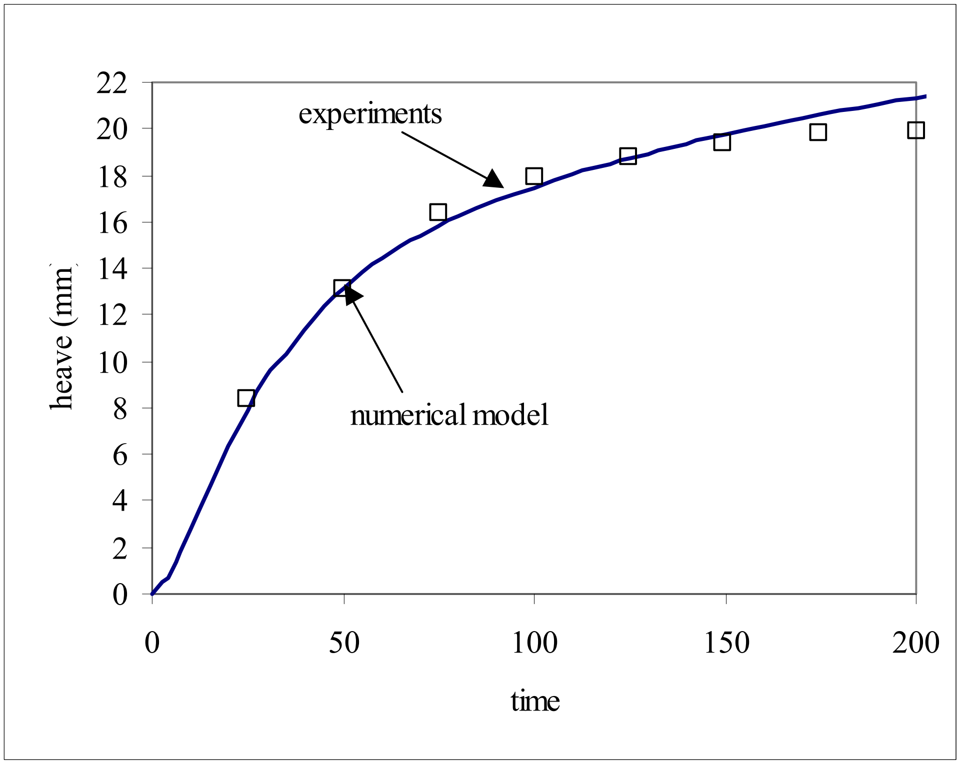


Figure 6.3 Comparison between Model and Experiments

Sensitivity Study

Hou, 1993 undertook an extensive assessment of the sensitivity of the parameters in Eq(5.2) and Eq(5.3), (i.e, $\dot{n}_m, T_m, \varepsilon, \gamma, \omega_r, \overline{\omega}$, and η). This was achieved by changing each best value in Table 6.2 by +10% and then by -10% while keeping the other best values fixed. Full results are reported by Hou, 1993. The key finding are

1. The heave results are not very sensitive to parameters $\varepsilon, \omega_r, \overline{\omega}$, and η.
2. Are sensitive to $\dot{n}_m$, T_m and especially γ.

This means that, in the proposed frost heave model, the sub-model which decides the porosity rate has more impact on heave results than the other sub-model which characterizes the unfrozen water content with temperature.

The porosity rate function characterizes individual soil samples and our sensitivity results indicate that frost heave is strongly dependent on the nature of the soil sample. This phenomenon is in accordance with real observations: sand does not have a frost heave problem, while clay and silt do. Due to differences in particle size, porosity, heat conductivity, etc, different clays or silts also have different degrees of frost susceptibility.

Numerical Considerations

For the above calculations, the domain was described using 30 control volumes and a 120 second time step. The CPU time required on the an Intel III processor is about 10 seconds for the 200 hours total analysis time. Also, if reasonable variations of different grid spacing and time step are chosen, the heave results are independent of them.

All the predicated results are calculated on a deforming grid and include the water convection term. Most frost heave models neglect the convection term and the effect of domain change on the heave calculation, (i.e, they use a fixed grid). The impact of the domain movement and water convection was investigated by Hou, 1993 (54). The effect of domain change on the heave calculation is significant (almost 10%) variation. The effect of the convection term is small.

CHAPTER 7

A THERMAL THAWING MODEL

Overview

As an extension of the one-dimensional frost heave model a thermal model that describes the development of the thermal field in a two-dimensional road cross-section during a freeze-thaw cycle is developed. This model focuses on predicting the thermal field in the sub-grade. The model is based on a finite element code and deal with non- rectangular geometries and non-constant thermal properties (e.g., different conductivities in different space regions).

Model Framework

From a scientific viewpoint the freezing and thawing in soil is a problem that is driven by events occurring at a microscopic scale (i.e., on the scale of the soil grains). Many of the soil freezing models focus on understanding these microscopic phenomena (e.g., modeling the formation of the so called ice lenses). This microscopic approach is limited to the investigation of small scale problems in simplified domains. An alternative engineering approach is to "lump" the micro-scale phenomena into a single variable [e.g., Blanchard and Fremond (1985)(1)] proposes a porosity rate term. In this way freezing/thawing processes in large-scale domains and the relevant engineering problems can be investigated. A full application of a Fremond-like model requires two steps. In the first step, for a given geometry and cooling conditions, the energy equation is solved and the porosity changes in the domain calculated. In the second step the strains resulting from porosity changes are incorporated in to a suitable constitutive equation and the stresses and displacements in the domain are calculated. This approach bears a strong resemblance to methods for calculating thermal stresses in metal solidification processes. In the current project, as a starting point, only the first step in the Fremond model will be developed. Our rational is that, with careful interpretation, much information can be gained from knowing the evolution of the temperature and porosity fields. Note an example of the full application can be found in the previous chapters of this report.

A simplified energy transport equation can be obtained from Eq. (3.16)

$$\frac{\partial T}{\partial t} = \nabla(\alpha \nabla T) - \frac{L}{C_p}\frac{\partial n\nu}{\partial t} \tag{7.1}$$

where

T is temperature (K)

L is latent heat (KJ/kg)

Cp is the specific heat (KJ/kg-K) a is diffusivity (m2/day)

n is porosity

ν is the pore liquid fraction

As noted in previous parts of this report, a key parameter in the model is the porosity rate function. At this point, in contrast to the more general model given above Eq (5.2) the simple model

$$\dot{n} = \beta A T e^{BT} \tag{7.2}$$

suggested by Fremond is used. In (7.2) A and B are material properties which can be measured in a laboratory test and $\beta = 1$ if $T < 0$ (freezing) or $\beta = -1$ it $T > 0$ (thawing).

Effect of Thawing around a Buried Culvert

The following example problem of a shallow sewer culvert is chosen, see Fig.7.1. This problem consists of a square culvert (0.5 m x 0.5 m) buried 1.25 m below the surface of 5 m x 5 m test section. The sides of the test section are insulated, the bottom temperature is fixed at the initial temperature of T = 10 ^{o}C. An initial porosity of n = 0.4 is assumed and the thermal properties used are as follows, L = 150 J/Kg, a = 0.11 J/m ^{o}C in silt and 0.1 J/m ^{o}C in sand, Cp=l J/Kg ^{o}C. The top temperature is given by the function

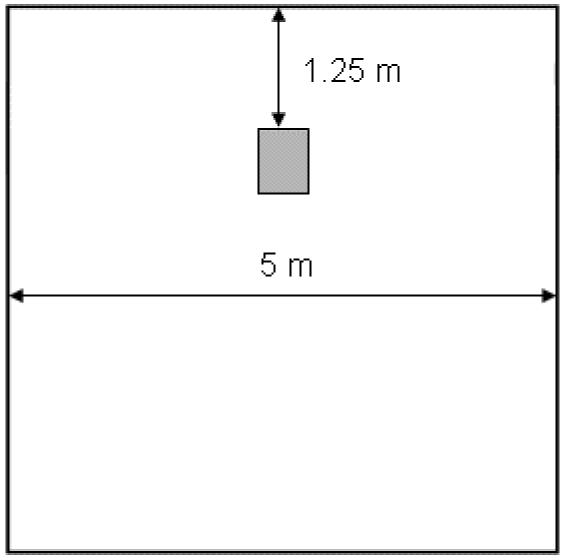


Figure 7.1 Test Geometry for Culvert Problem

$$T(t) = \begin{cases} 10 - \dfrac{2t}{3} & \text{if } t < 30 \text{ days} \\ -10 & \text{if } 30 \text{ days} \le t \le 90 \text{ days} \\ -10 + \dfrac{2(t-90)}{3} & \text{if } t > 90 \text{ days} \end{cases} \tag{7.3}$$

and the pore water content and temperature relationship is given by

$$\nu = \begin{cases} 1 & \text{if } T > 0 \\ \dfrac{(T+2)^2}{16} & \text{if } -2 \le T < 0 \end{cases} \tag{7.4}$$

During the freeze-thaw cycle, the inside surface temperature of the culvert is fixed at the same temperature as the upper surface ,Ta. The rate of heat transfer, per unit area of the culvert, into soil is given by

$$Q = h(T_s - T) \tag{7.5}$$

In the current work, we will investigate the effect of the heat transfer coefficient, h, in Eqn. (7.5). In particular its effect on the movement of the freezing front (i.e., the T = 0 °c contour) will be examined. Two cases will be looked at. In the first case, h = 2 J/m^2-day corresponding to a culvert in good thermal contact with the soil. In the second case, h = 0.2 J/m^2-day corresponding to a case where the culvert is insulated. In each case predictions for the position of the freezing front with time (based on a 20x20 finite element grid) can be obtained. Of particular interest in these predictions is the extent of the region of "unstable soil" (i.e. thawed-previously frozen soil) around the culvert. Due to the melting of the water, which migrated into the freezing zone, this region will, if drainage is not sufficient, be over-saturated and lose its ability to support weight. If the region is large, the culvert may "shift" upwards driven by buoyancy forces. Figure 7.2 compares the predictions for the maximum penetration of the frost line at t = 105 days (the solid line) with the frost penetration after thawing has started at t = 120 days (the dashed line) when h = 2 J/m2-day. The region in the vicinity of the culvert, "sandwiched" between the dashed and full lines, represents a region of unstable soil. In this case, the region is relatively large indicating a potential problem of culvert lift. Figure 7.3 shows the equivalent result for the case of the insulated culvert (h = 0.2 J/m2-day). In this case, the region of unstable soil in the vicinity of the culvert is noticeably reduced. Shifting of the culvert may not occur in this case.

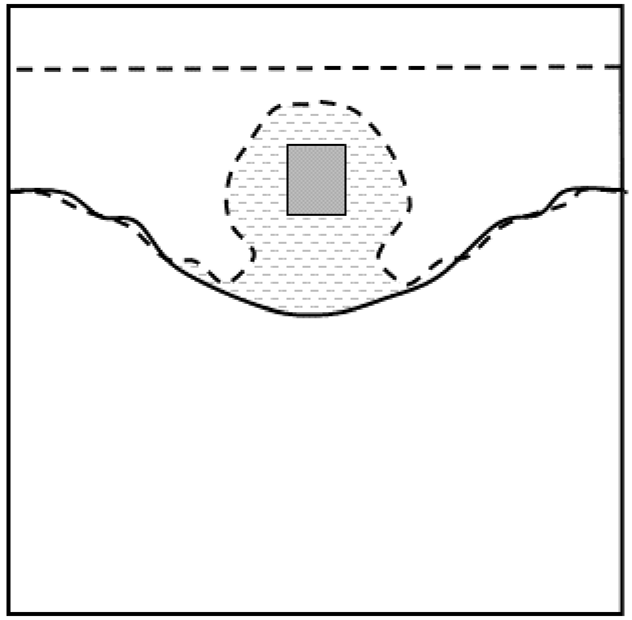
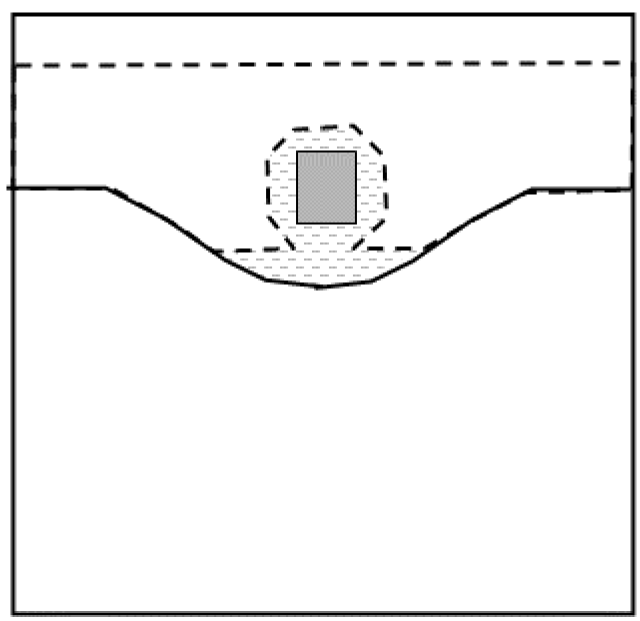


Figure 7.2 Frozen-Thawed Region when h = 2 J/m^2-day

Figure 7.3 Frozen-Thawed Region when h = 0.2 J/m^2-day



CHAPTER 8

SUMMARY AND RECOMMENDATIONS

Summary

The objective of this study has been to investigate the lumped parameter frost heave model developed by Blanchard and Fremond. In this respect, the key contributions have been:

1. The development of general transport governing equation based on an averaging multi-phase analysis, i.e, Eq.(3.12) and Eq.(3.17).
2. The application of an efficient numerical phase change algorithm for the solution of the lumped parameter frost heave model. In this numerical solution, the heaving deformations of the domain and convection effects are fully accounted for.
3. The development of a simplified two-dimensional thermal model that can be used to investigate the effects of freeze-thaw cycles on buried infrastructure.

Recommendations

Towards developing an understanding of how thawing and freezing cycles impact transportation infrastructure there are significant advantages in building a lumped 'macro-scale' model, especially when dealing with two- or three-dimensional problems, where it is extremely difficult to measure micro-process parameters (e.g. ice velocity and its direction). This work has shown that a lumped porosity model can generates accurate comparison with experiments and is feasible in gaining insights into the effect of freeze thaw on buried infrastructure. It is recommended that further work is undertaken to develop user friendly software based around the lumped porosity model and thereby extend its application across a wide range of transportation infrastructure problems. In the mean time it is recommended that practical construction methods and non frost susceptible materials be utilized, see Svec and Gallagher (2002). A synthesis of innovative construction practices should also be completed.

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