Access to Destinations

Arterial Data Acquisition and Network-Wide Travel Time Estimation (Phase II)

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16. Abstract (Limit: 250 words)

The objectives of this project were to (a) produce historic estimates of travel times on Twin-Cities arterials for 1995 and 2005, and (b) develop an initial architecture and database that could, in the future, produce timely estimates of arterial traffic volumes and travel times. Our Phase I field study indicated that on arterial links where both the demand traffic volume and the signal timing are known, model-based estimates of travel time that are on average within 10% of measured values can be obtained. Phase II of this project then focused on applying this approach to the entire Twin Cities arterial system. The Phase II effort divided into three main subtasks: (1) updating estimates of demand traffic volume obtained from a transportation planning model to make them consistent with available volume measurements, (2) collecting information on traffic signal locations in the Twin Cities and compiling this into a geographic database, and (3) combining the updated traffic volumes and signal information to produce link-by-link peak-period travel time estimates. The traffic volume update took as inputs the predicted volumes generated by a traffic assignment model and measured average annual daily traffic from automatic traffic recorders, and gave as output updated estimates of the traffic volumes for links lacking automatic traffic recorders. A request to state, county and municipal agencies in the seven-county metro area produced Information on approximately 2,900 traffic signals. Estimated arterial travel times for the morning and afternoon peak periods for 1995 and 2005 were then computed and sent to other components of the Access to Destinations effort.

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Access to Destinations:
Arterial Data Acquisition and Network-Wide Travel Time Estimation (Phase II)

Final Report

Prepared by:

Gary A. Davis
John Hourdos
Hui Xiong
Ted Morris

Department of Civil Engineering
University of Minnesota

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Executive Summary

The objectives of this project were to (a) produce historic estimates of travel times on Twin-Cities arterials for 1995 and 2005, and (b) develop an initial architecture and database that could, in the future, produce timely estimates of arterial traffic volumes and travel times. Estimating traffic volumes and travel times is relatively straightforward on roads with a spatially dense placement of traffic sensors, but becomes more difficult where sensors are sparse or nonexistent. However, our Phase I field study indicated that on arterial links where both the demand traffic volume and the signal timing are known, model-based estimates of travel time that are on average within 10% of measured values can be obtained. Phase II of this project then focused on applying this approach to the entire Twin Cities arterial system. The Phase II effort divided into three main subtasks: (1) updating estimates of demand traffic volume obtained from a transportation planning model to make them consistent with available volume measurements, (2) collecting information on traffic signal locations in the Twin Cities and compiling this into a geographic database, and (3) combining the updated traffic volumes and signal information to produce link-by-link peak-period travel time estimates on Twin Cities arterials for 1995 and 2005.

The traffic volume update took as inputs the predicted volumes generated by a traffic assignment model and measured average annual daily traffic from automatic traffic recorders, and gave as output updated estimates of the traffic volumes for links lacking automatic traffic recorders (ATRs). Original computer code for computing the required covariance matrices and performing the update was developed by this project and runs on a University of Minnesota supercomputer. Compiling the information on signal locations and timings began with a request to state, county and municipal agencies in the seven-county metro area for information on the locations and timings of their signals. Over 3,000 possible signal locations were identified, and for some 2,900 information was sufficient to include them in a GIS database developed by this project. Integrating this information with the planning model representation of the Twin Cities network from 1995, and to a lesser extent 2005, proved to be a major challenge due to the use by the model's use of generalized representations for many arterial links. However, estimated arterial travel times for the morning and afternoon peak periods for 1995 and 2005 were computed and sent to other components of the Access to Destinations effort, and the report includes several illustrative travel time contour maps developed from these estimates. The report ends with some suggestions on how to leverage this work to produce a system for monitoring arterial traffic volumes and travel times in the Twin-Cities.

Chapter 1: Introduction

1.1 Background

This report describes work done for the project Access to Destinations: Arterial Data Acquisition and Network-Wide Travel Time Estimation-Phase II. The main objectives of this project were to (1) produce estimates of travel times on arterial and collector roads in the Twin Cities metropolitan area for the years 1995 and 2005, and (2) develop an initial architecture and database to support ongoing monitoring of arterial traffic volumes and travel times in the Twin Cities. Since the 1970's, effective real-time management of urban freeways has been a goal of both state and federal departments of transportation. This has led to the development of freeway monitoring systems in many U.S. urban areas, where the installation of inductive loop detectors and video surveillance now provide detailed information on freeway traffic conditions, from which it is possible to produce plausible estimates of the travel times on sections of the freeway system. In fact, these surveillance system data are the backbone of many driver information systems. In contrast to urban freeways, the monitoring of conditions on arterials is at a much less-developed stage. Although inductive loop technology is often used to control signalized intersections, collecting and storing data from these loops is the rare exception rather than the rule. A similar situation applies to the use of video surveillance at intersections. In part, this is due to the fact that while an urban freeway system is usually managed solely by a state department of transportation, an urban area's arterial system is usually managed by a number of county and municipal units. It is also due to the fact that the total size of an urban area's arterial system is generally much larger than that of its freeway system. This means that while it is in principle possible to construct historic estimates of travel times on the Twin Cities freeway system by using archived inductive loop detector data, producing similar estimates for Twin Cities arterials is not at present possible.

1.2 Phase I Overview

In the absence of a system for automatic collection and storage of traffic data on Twin Cities arterial system, historical estimates of travel time must be based on some form of prior knowledge about how arterials perform, combined with what limited historic data do exist. The primary objective of Phase I of this project was to identify and evaluate parametric models for making default estimates of travel times on arterial links, using information typically available from a transportation planning model. The chosen method of evaluation was to compare travel time predictions generated by the models to field measurements. A review of the literature revealed several candidate models, including the Bureau of Public Roads (BPR) function, Spiess's conical volume delay function, the Singapore model, the Skabardonis-Dowling model, and the *Highway Capacity Manual's* (HCM) model. A survey of Twin-Cities traffic agencies indicated that it would be difficult to obtain, from existing data sources, a representative sample of arterial links with all data needed to evaluate the candidate models, so it was decided to conduct a field study to collect the needed data. In a first pilot study, average travel times on an arterial link were measured using (1) a combination of spot speed data to estimate mean free-flow travel times and intersection delay measurements to estimate average waiting time, and (2) a floating car method. The floating car method turned out to be sensitive to the relative fraction of runs where the floating car was delayed by a red signal indication, and it was not possible to

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obtain enough runs to reliably estimate this fraction. A comparison of travel time models using the data collected with method (1) indicated best performance by the HCM model, the worst performance by the BPR and conical volume-delay models, with the remaining two models being slightly worse than the HCM model.

To maximize the number of data collection sites within the Phase I project's resource constraints it was then decided to evaluate a license plate matching method for collecting travel time data, which could be carried out by only two field personnel. A second pilot study comparing the travel times measured using the license plate method to travel times measured from video revealed a tendency for the license plate method to under-sample vehicles stopped by red signal indications, and hence to underestimate average travel times. However, by using a mixture decomposition method to estimate mean travel times for stopped and non-stopped vehicles, together with an independent estimate of proportion stopping, an estimation method that substantially eliminated this bias was developed.

The license plate method was then applied to a sample of 50 arterial links located in the Twin Cities seven county metropolitan area, to obtain measurements of average travel time. Also obtained were the lengths of each link, measurements of traffic volume, and signal timing information. Default values for model parameters were obtained from the Twin Cities planning model's database. Using network default parameters, we found that the BPR and conical volume-delay models produced mean average percent errors (MAPE) of about 25%, while the Singapore and Skabardonis-Dowling models, using maximal site-specific information, produced MAPE values of around 6.5%. As site-specific information was replaced by default information, the performance of the latter two models deteriorated, but even under conditions of minimal information the models produced MAPE values of around 20%. A cross-validation study of the Skabardonis-Dowling model showed essentially similar performance when predicting travel times on links not used to estimate default parameter values.

1.3. Brief Description of Phase II Effort

The main findings from Phase I were that the most reliable model-based estimates of travel time required knowing (1) the demand traffic flow on an arterial segment, and (2) the timing of a signal at the segment's end. Phase II of this project then involved applying the results from Phase I to produce estimates of arterial travel times in the Twin Cities of the years 1995 and 2005. The overall effort in Phase II was thus divided into two major thrusts (1) developing a method for updating traffic volume estimates produced by a transportation planning model to make them consistent with the limited existing data from arterials and (2) constructing a database recording the location and timings of signalized intersections in the Twin Cities. Chapter 2 of this report begins by describing a method, developed for this project, for updating the link volume estimates produced by a planning model using data from automatic traffic recorders. Chapter 2 then concludes by describing production of a set of default travel time estimates computed without knowledge of signal location or timing. Chapter 3 describes the construction of a signalized intersection database for the Twin Cities metro area and its linkage with metro area network models. This chapter also describes computation of updated travel times which use the available signal information. Chapter 4 then gives our conclusions and suggestions for extending this work.

Chapter 2: Estimating System-wide Urban Traffic Volumes and Default Travel Times on Arterials

2.1 Introduction

Traffic volume counts are important and widely used measures of travel demand. Estimates of traffic volumes for particular periods within a day, such as the morning peak period, are used in assessing capacity and level of service, in selecting and implementing traffic control devices, and in traffic impact analyses. Traditionally, the engineer or planner needing a volume estimate on an urban arterial had three options. The happiest possibility was that the road of interest happened to have a permanently installed automatic traffic recorder (ATR). However, ATRs are found on only a small number of road segments. Another option was to use a volume forecast prepared using a transportation planning model. Finally, the estimate could be based on a short traffic count conducted on the road of interest. This count could be done manually or with a portable traffic counter, but for practical reasons this count would be made over a relatively small number of days (one to three being typical). The results of this short count would be averaged and this average adjusted to account for biases resulting from seasonal or within-week trends. The adjustment terms would have been estimated from trends identified using a hopefully representative ATR. The short count could be done specially for a particular project or it could have been part of a jurisdiction's traffic monitoring program.

In this chapter we will describe a hybrid method for obtaining estimates of annual average traffic volumes, for specified within-day periods, on urban arterials. In essence, our method uses volume measurements from ATRs to update the forecasts prepared by a planning model. In what follows we will first review traffic volume estimation in more detail, and then outline the theoretical basis for our method. We will then present a validation study using data from the Twin Cities metropolitan region, followed by a discussion regarding the possible reasons of limited improvement. At last, the procedure to estimate default arterial travel times will be illustrated.

There are two methods to obtain traffic volume information for a time period: one is based on the traffic data collection, and the other is via the traffic assignment models. Traditionally, traffic count data are collected by ATRs and Short Period Traffic Counters (SPTC). ATRs not only provide accurate continuous traffic counts on ATR links, but also establish day-of-week and seasonal factors to adjust traffic counts on SPTC links to estimate their Annual Average Daily Traffic (AADT). SPTC links are selected to provide the required geographic coverage, whose traffic counts are typically collected for 24-48 hours every 2-4 years, and must be adjusted to convert to AADT. Once AADT is available, it would need to be "scaled down" by multiplying another factor to estimate the traffic volume for a desired within-day period.

Adjustments for trend bias are found by assigning the site of interest to a factor group, a set of road segments having similar trends and containing at least one ATR. Studies concerning factor group assignment have been extensively conducted. It was found that the error of AADT estimation due to the wrong assignment of SPTC counts into factor groups is dominant and potentially very substantial. Davis (1997) reviewed the accuracy of estimates of mean daily traffic. He pointed out the insufficiency of 24- or 48-hour coverage counts recommended in past

guidance and suggested using the data-driven Bayesian method for factor group assignment. Sharma and Allipuram (1993) described Bayesian methods that provided an average accuracy of less than 20% error most of time, given carefully selected 14 sampling days. Davis and Guan (1996) developed a Bayesian estimator of mean daily traffic and found the classification method may require at least 14-day samples to reliably assign the coverage counts to the correct factor group.

McCord et. al (2003) and Jiang (2005) conducted studies utilizing image-based data extracted from high resolution air photos, satellite images, and LiDAR (light detection and ranging) data to estimate AADT. They showed that a small amount of image-based data could provide improved accuracy over the traditional ground-based traffic count methods and reduce the number of SPTCs. Although image-based data are increasingly easy to obtain, the batch processing of large image files is one of the biggest obstacles to applying this method at a network level.

The second method is using traffic assignment models, which model the travelers' path selection between origins and destinations. Transportation analysts can estimate the traffic flows on all links in a network, given (1) an origin-destination (OD) table; (2) a network geometry, such as start/end nodes, link length, free-flow speed (FFS), and capacity; (3) link performance functions; and (4) a traffic assignment model. The accuracy of assigned link flows (annual average for a specific period) is influenced by numerous factors, such as the zonal partitioning of the region and demand estimates in the OD table.

There is limited published work evaluating the above-mentioned method. Florian and Nguyen (1976) tested the validity of equilibrium traffic assignment using 1970 network data of the City of Winnipeg, Manitoba, Canada. Their results showed "surprisingly good" concordance between predicted and observed volumes. Cascetta (1989) described a model taking into account stochastic fluctuations of demand and reported the results of comparison of stochastic dynamic traffic assignment (STODYN) average flows with Stochastic User Equilibrium (SUE) and observed flows using data from two small cities in Italy. He showed the similarity of probit-based STODYN and SUE models in terms of estimation percent root mean square errors, which were both 32% for the Town of Parma, and 50% and 48% for the Town of Foggia.

In practice, urban traffic monitoring has been concentrated on freeways rather than arterial roads. With thousands of sensors available in a typical freeway system of a big urban network (e.g. Twin Cities network has 3,780 freeway inductive loop detectors), the information is adequate for traffic estimation and control purposes. However, the traffic data collection on arterials is relatively lacking and far from sufficient to provide a network level "ground truth". Within the last decade, there have been studies on development of arterial monitoring (or performance measurement) system (e.g. Nee and Hallenbeck 2001; Petty et al. 2005; FHWA 2007; Liu et al. 2009). So far, because of cost constraints, network level arterial traffic monitoring systems have not been developed to a level similar to freeway systems.

2.2 Model

2.2.1 Gaussian Process (GP)

Let X denote a vector containing a network's link flows, partitioned into unobserved and observed components. That is $X = \begin{bmatrix} X_U \\ X_A \end{bmatrix}$ and for large populations X is approximately multivariate normal (Dial 1971; Sheffi 1985) with mean vector $\mu = \begin{bmatrix} \mu_U \\ \mu_A \end{bmatrix}$ and covariance matrix

$$Q = \begin{bmatrix} Q_{U,U} & Q_{U,A} \\ Q_{A,U} & Q_{A,A} \end{bmatrix}.$$

If x_A denotes actual observations for the observed component X_A, then the conditional distribution for the unobserved links will be approximately multivariate normal, with mean

$$\hat{X}_U = \mu_U + Q_{U,A}Q_{A,A}^{-1}(x_A - \mu_A) \tag{2.1}$$

and covariance is given by

$$\hat{Q}_{U,U} = Q_{U,U} - Q_{U,A}Q_{A,A}^{-1}Q_{A,U}. \tag{2.2}$$

If we take μ_U and μ_A as output from a traffic assignment model, then it is apparent that in Eq. (2.1) and Eq. (2.2) that the calculation of Q is key to implementing GP. Figure 2.1 shows a flow chart of our proposed method.

2.2.2 Covariance Matrix Calculation

SUE is useful in transportation planning because it takes into account drivers' dispersion in route travel time perception, which more realistically represents travelers' decision. Sheffi (1985) described how to calculate a link flow covariance by modeling the covariance elements as products of OD demands and route use probabilities. Assuming that a unique SUE exists, the path flow **F** can be approximated by a multivariate normal distribution:

$$F \sim MVN(f, \Sigma), \tag{2.3}$$

where f = path flow mean and Σ = path flow covariance.

If the link-path incidence matrix Δ is known, it can then be shown that the link flows X also approximately follow a multivariate normal distribution:

$$X \sim MVN(f\Delta^T, Q), \tag{2.4}$$

where $f\Delta^T$ = link flow mean and Q = link flow covariance ($\Delta\Sigma\Delta^T$).

NETWORK INFORMATION

OD TABLE

ATR VOLUMES

SUE TRAFFIC ASSIGNMENT

SUE LINK FLOWS

LINK FLOW COVARIANCE MATRIX CALCULATION

LINK FLOW COVARIANCE MATRIX

GAUSSIAN PROCESS

ESTIMATED LINK FLOWS
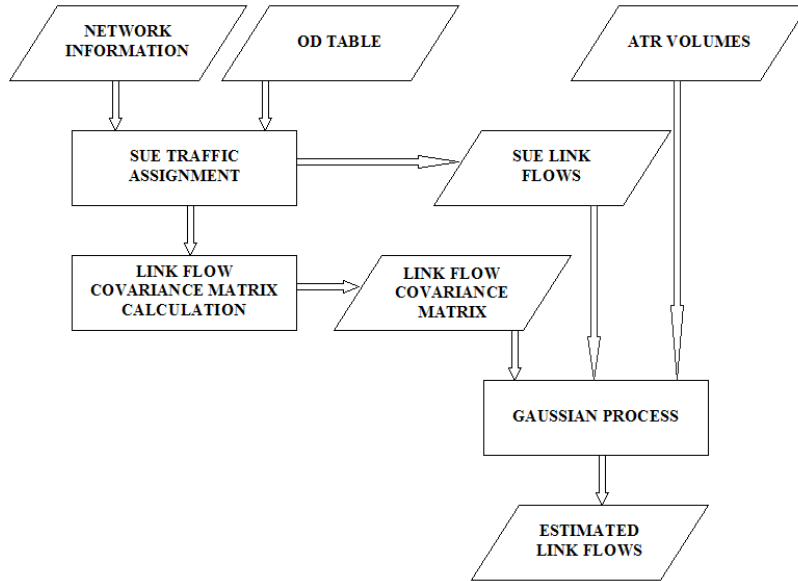
Figure 2.1 Flow Chart of Proposed Method.

The pre-specification of Δ and the calculation of Σ is theoretically possible for a real transportation network with thousands of links, but unfortunately will take prohibitively long. However, if route choice is governed by a logit model, then a variant of Dial's (1971) assignment model can be used to compute the required covariance matrix without path enumeration (Davis and Nihan 1993; Davis 1994). The simplest version of this method occurs when the OD demands are treated as Poisson random variables.

2.3 Data

In this study, we focused on Twin Cities 1995 and 2005 planning networks. Their main features are summarized in Table 2.1. The 2005 network is an expanded version of 1995 network, but we limited our study to the seven-county Metropolitan area covered by both networks. The Metropolitan Council defines AM and PM peaks as 6:30AM to 7:30AM and 3:30PM to 4:30PM respectively.

TABLE 2.1 Main Features of 1995 and 2005 Twin Cities Transportation Networks

	1995 AM	1995 PM	2005 AM	2005 PM
Nodes	7,776		9,689	
Links (unobserved arterial links)	20,486 (12,305)		25,378(13,182)	
Transportation Analysis zones	1,200		1,632	
Non-zero demand OD pairs	197,920	247,597	814,070	842,053

Arterial links were identified by assignment group codes in the 1995/2005 network files. In the 1995 network, arterial links were those with assignment group code =5 (divided arterial, includes expressways), =6 (undivided arterial), and =7 (collector). In the 2005 network, they were those with assignment group code =5 (divided arterial, excludes expressway), =6 (undivided arterial), =7 (collector), and =15 (expressway).

Four sets of ATR count data were extracted from ATR data files provided by Minnesota Department of Transportation (MnDOT): 1996/2005 AM/PM peaks (1995 ATR data were not available so 1996 data were used instead). The 1995 and 2005 Metropolitan Council OD tables represent demands on typical weekdays and divided a day into multiple durations. To be consistent with OD tables, weekend and holiday ATR counts were removed and remaining counts were summed and divided by total number of regular days. In 1996, there were 146 ATRs in good condition (working well continuously for the whole year). During 2001, some ATRs were believed redundant and then removed by MnDOT. In 2005, there were only 72 ATRs. All ATRs were geo-coded manually. On the basis of above procedures, we obtained the observed ATR counts.

Figure 2.2 and Figure 2.3 show the Twin-Cities networks and ATR locations.
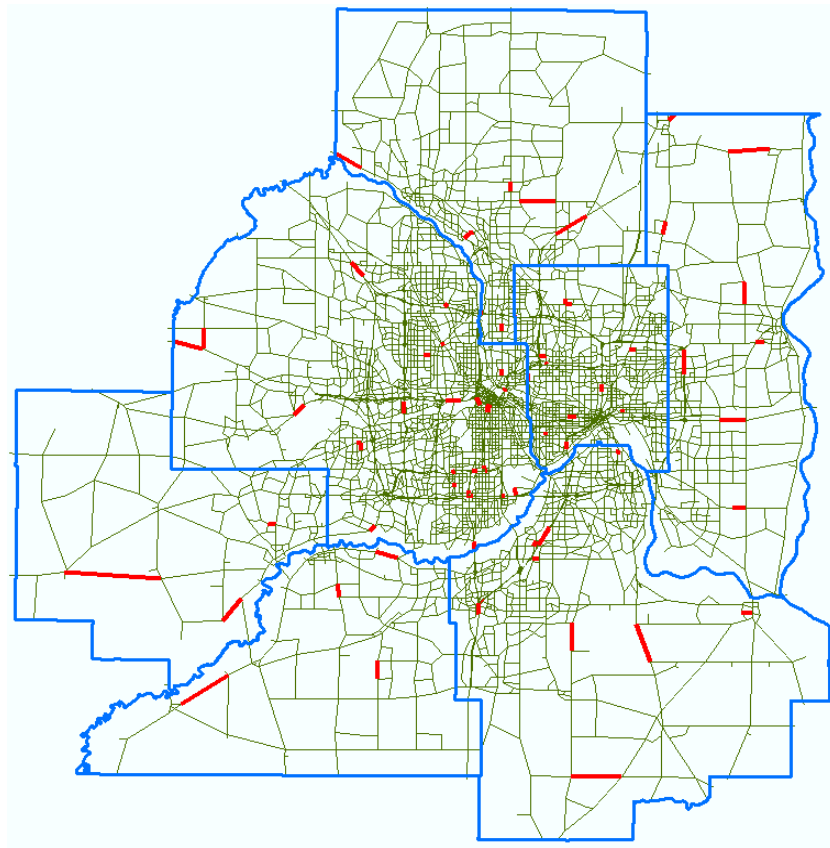
Figure 2.2 1995 Twin Cities Network and 1996 ATR Locations.
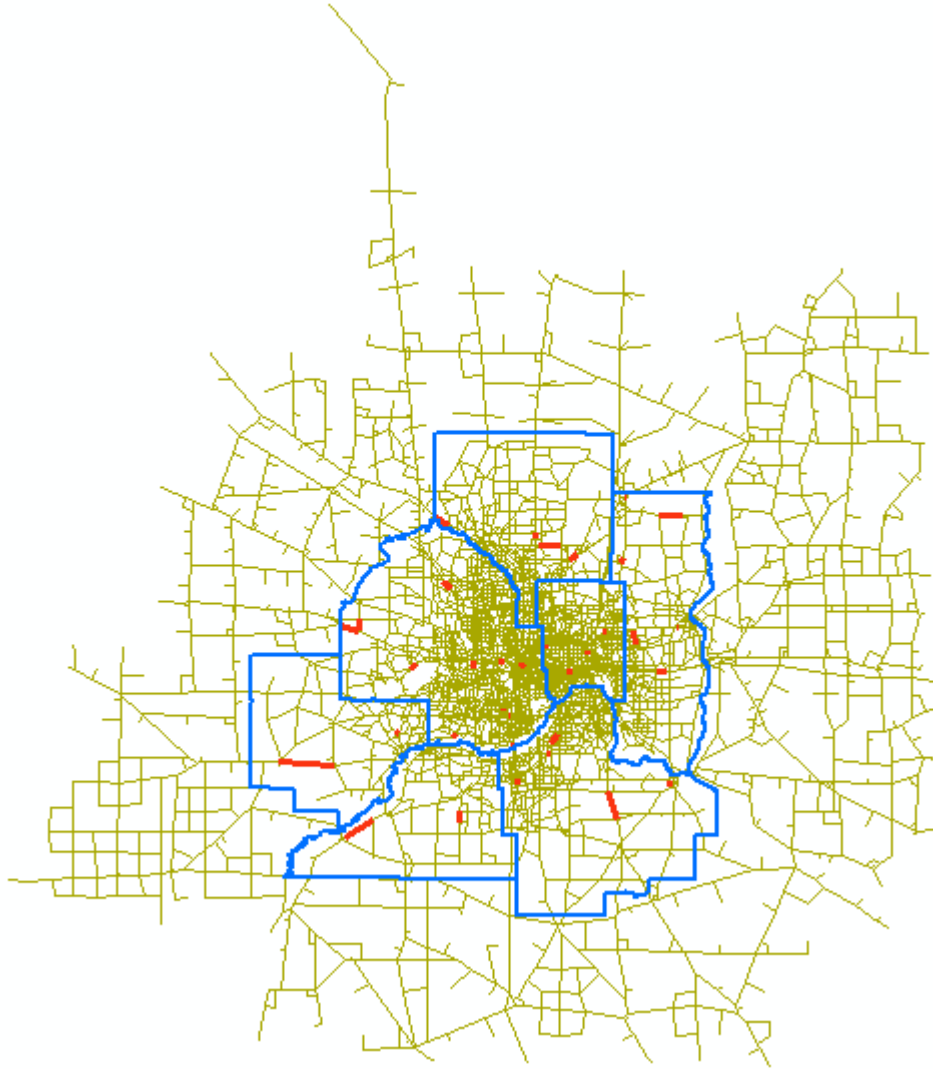
Figure 2.3 2005 Twin Cities Network and 2005 ATR Locations.

2.4 Results

Our method was coded in FORTRAN 77 and ran under the Linux system of IBM Blade Center at the University of Minnesota Supercomputer Institute. With network and ATR data available for the years of 1995 (1996) and 2005, we performed the SUE assignment and noticed there existed differences between the sum of observed ATR flows and the sum of assigned flows on ATR links. A simple calibration was implemented by multiplying all OD demands by a factor for each of four peak periods. Then we calculated the link flow covariance and updated unobserved arterial flows via GP for AM and PM peak periods. Figure 2.4 gives the scatter plots of unobserved arterial link flows, in which the X-axis represents the SUE flows (before GP) and Y-axis represents the updated flows (after GP). The dispersion about 45° line shows the updated links have means equal to SUE flows.
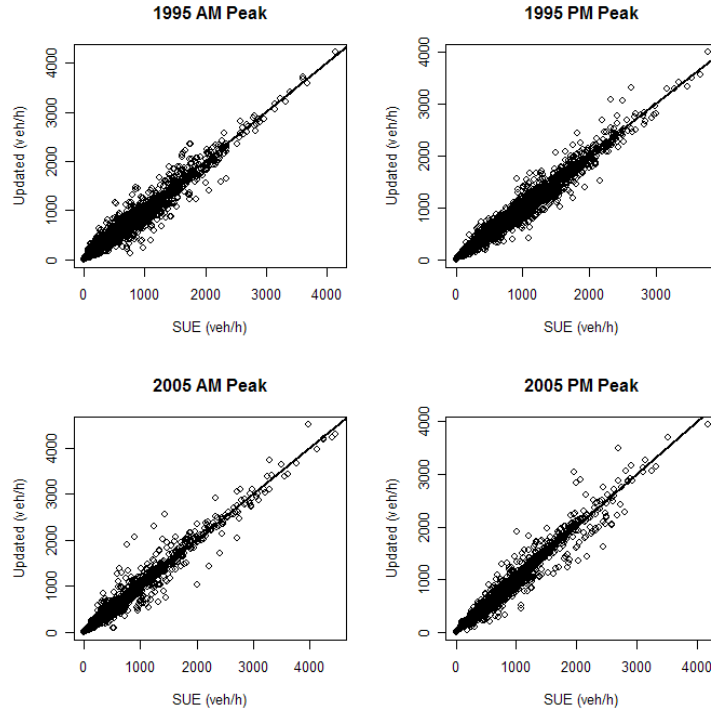
Figure 2.4 Unobserved Arterial Link Flow Scatter Plots (SUE vs. Updated).

An informative model assessment should involve comparing estimated and measured flows for unobserved links. In this study, except for observed ATR counts, there were no other field-collected continuous data available. So it is impossible to evaluate the accuracy improvement of arterial flow estimates directly. To test the performance of our model, we implemented a procedure called Leave-one-out Cross-validation (LOOCV) using ATR counts. The procedure is as follows: (1) use a single ATR observation as the validation data, and the remaining ATR observations as the training data (in GP, only these links are viewed as observed); (2) perform the GP to update the SUE flow of that single link; (3) repeat (1) and (2) until all ATR links have been used for validation counts.

If estimated ATR counts are equal to observed ATR counts, in their scatter plot, all data points should locate on the 45° line. Figure 2.5 shows the 1995/2005 AM/PM peak scatter plots of observed vs. SUE link flows. The obvious dispersion indicates low accuracy. After GP, the plots of observed vs. updated (see Figure 2.6) display the similar scattered patterns as before, meaning improvement is very limited, especially for the year of 2005.

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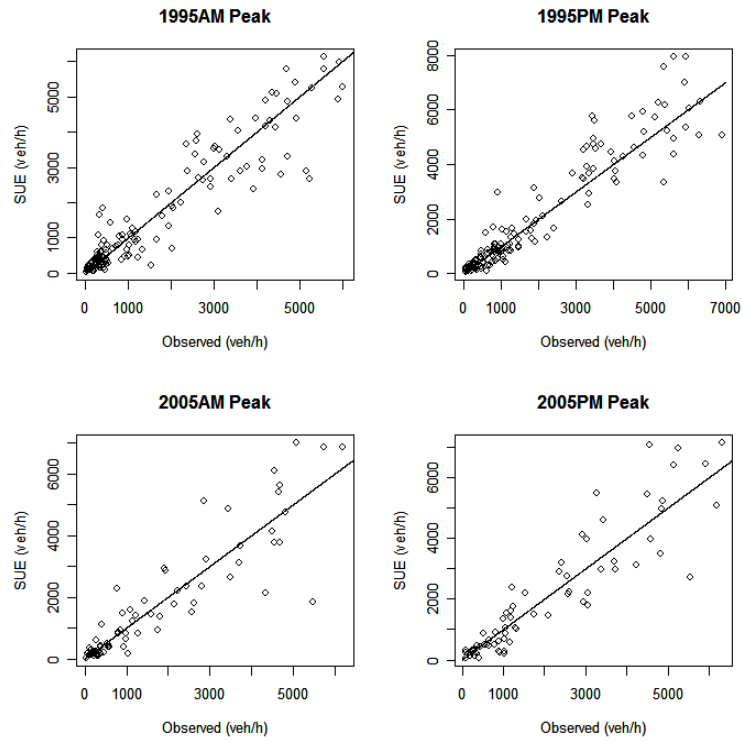
Figure 2.5 ATR Link Flow Scatter Plots (Observed vs. SUE).
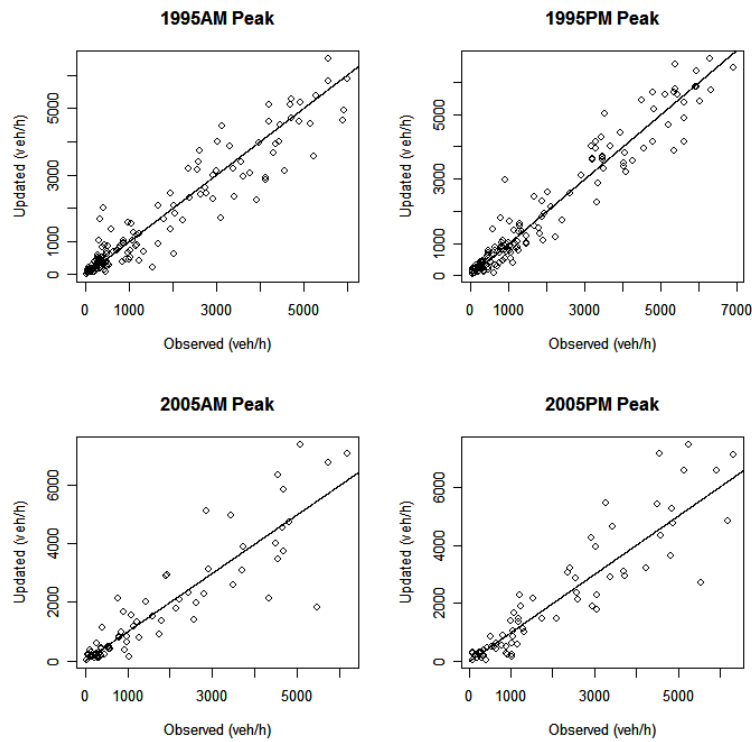
Figure 2.6 ATR Link Flow Scatter Plots (Observed vs. Updated).

To measure the over-all estimation accuracy, we adopted two criteria: the mean absolute percentage error (MAPE) and the root of mean square error (RMSE). MAPE and RMSE are defined as:

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\frac{|O_i - E_i|}{O_i} \times 100\% \ . \qquad (2.5)$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(O_i - E_i)^2}{n}} \qquad (2.6)$$

where O_i stands for observed value and E_i stands for estimated value; and

n is the number of observations/estimates.

A lower value for MAPE or RMSE thus indicates better accuracy.

Table 2.2 is a summary of MAPE values and Table 2.4 is a summary of RMSE values. It can be seen that: (1) the 1995 PM peak has a noticeable improvement, especially its RMSE (reduced about 30%); (2) the 1995 AM peak has a very limited improvement; (3) the 2005 AM/PM peaks do not have any improvement. Actually, the LOOCV estimates are, on average, slightly worse than SUE.

TABLE 2.2 Summary of MAPE Values

	1995 AM	1995 PM	2005 AM	2005 PM
SUE	48.43%	37.10%	42.15%	37.26%
Updated	47.29%	33.67%	43.50%	38.00%

TABLE 2.3 Summary of RMSE Values

	1995 AM	1995 PM	2005 AM	2005 PM
SUE	603.65	699.17	812.71	807.33
Updated	546.27	489.12	846.38	832.55

2.5 Default Travel Time Estimation

Since the signal timing database was being developed in parallel with the volume updating effort, an initial set of default travel time estimates was computed using the Bureau of Public Roads (BPR) function instead of the Skabardonis-Dowling model. The BPR function has the form

$$TT = \frac{3600L}{FFS}(1 + a(\frac{X_0}{C})^b),$$ (2.7)

where TT= travel time (seconds),

L=link Length (miles)

FFS=free flow speed (miles per hour),

C= link capacity (vehicles per hour),

a=0.15,

b=4, and

X_0=link flow (vehicles per hour).

Estimates of the variance in the TT estimates were also computed using the relation

$$Var(TT) = Var(E(TT \mid X_0)) + E(Var(TT \mid X_0)).$$ (2.8)

Assuming FFS is constant, the first term of right-hand-side of Eq.(2.8) is

$$\left(\frac{d}{dx}\left[\frac{3600L}{FFS}(1 + a(\frac{X_0}{C})^b)\right]\Bigg|_{X_0=x_0}\right)^2 Var(X) = \left(ab\frac{3600L}{FFS}\frac{x_0^{b-1}}{C^b}\right)^2 Var(X_0).$$ (2.9)

The data collection sites in the Phase I field study were sampled from divided/undivided arterial roadways with signals. When estimating the default travel times, site-specific signal information was not available, so the default estimates are based on two assumptions: (1) all divided/undivided arterial links and expressway links have one and only one signal on them; and (2) collectors do not have signals. For the collectors, the second term of right-hand-side of Eq.(2.8) is not available; for those non-collector arterials, it is obtained from the field data, which is equal to 206.7, the variance of error terms obtained by applying the BRP function to the field study data. Since the BPR function does not explicitly consider signals, we assumed the signal delay accounted for the aforementioned difference and then added the mean of that (15.0 seconds) to Eq.(2.7) to estimate the default travel times.

We treated collectors and other arterials (non-collectors) separately and created totally eight spreadsheets to summarize 1995/2005 AM/PM peak arterial travel time estimates. Figure 2.7 is a snapshot of 1995 AM peak non-collector arterial travel time spreadsheet.

Figure 2.7 1995 AM Travel Time Spreadsheet (Non-collector Arterials).

The fields of spreadsheet in Figure 2.7 are defined as follows:

C1: Link ID in 1995 network

C2: updated SUE link flow via GP

C3: BPR travel time

C4: Estimated link travel time (= BPR travel time + 15.03)

C5: Variance of link flow

C6: Variance of travel time due to the uncertainty of link flow

C7: Variance of travel time due to the default signal delay

C8: Standard deviation of travel time (square root of C6+C7)

Chapter 3: Establishing a Metro-wide Database of Arterial Control Information

This chapter is organized into three sections which describe the results and methods used for developing the metro-wide database of arterial control information. The first section summarizes the location and warrant data information harvested to develop the geo-spatial database of traffic signals. Section 3.2 describes the current design and implementation of the database. The last section, Section 3.3, then describes the steps and methods used to develop the database. The latter section also provides useful information for future development of similar databases or for enhancing the current database to support other planning and traffic operations and management research.

3.1 Identification of Arterial Control Data

As noted earlier in the report, the establishment of signalized intersection locations alone can improve arterial travel time estimates by a significant margin. Therefore, the first task for developing the database herein consisted of harvesting such information in addition to signal turn-on dates (or their date of warrant) from various jurisdictions. At the onset of the project Mn/DOT provided a spreadsheet of their signalized intersection locations and turn-on dates on record at that time. Traffic operations staff from each of the seven counties was contacted to supply similar information, in whatever format could be provided. Typically this was in a spreadsheet form. Some jurisdictions were able to provide geo-spatial data files as well. Next, several municipalities were contacted: first, the major cities of Saint Paul and Minneapolis, followed by most of the ring suburbs (Edina, Saint Louis Park, Roseville, Golden Valley, Richfield & Bloomington, Brooklyn Center/Crystal/Robinsdale, Egan, Woodbury,) and finally some of the outer ring suburbs (Eden Prairie, Blaine, Minnetonka, Waconia, Plymouth, Vadnias Heights). Many municipalities reported that they were not responsible for the operations of their signals. Furthermore, a list by Hennepin County (without warrant dates) indicated municipally operated signalized intersection locations within the county. The total number of signalized intersections that were geo-located from these data stores was 3,074 signals. Note that approximately thirty of the city of Bloomington signals, primarily on collector arterials, were not joined in the travel time network due to missing locations and location errors. As of this writing, these have now been rectified and can be used for future work. Table 3.1 summarizes the distribution of geo-located signalized intersections completed for the project while Figure 3.1 displays a map of the metro area GIS with the signalized intersections color and shape coded. Signals marked with a triangle belong to counties, circles mark Mn/DOT owned signals, while squares mark cities and townships. The processing of the various forms of data in order to geo-reference signalized intersections will be described in detail in the final section of the chapter.
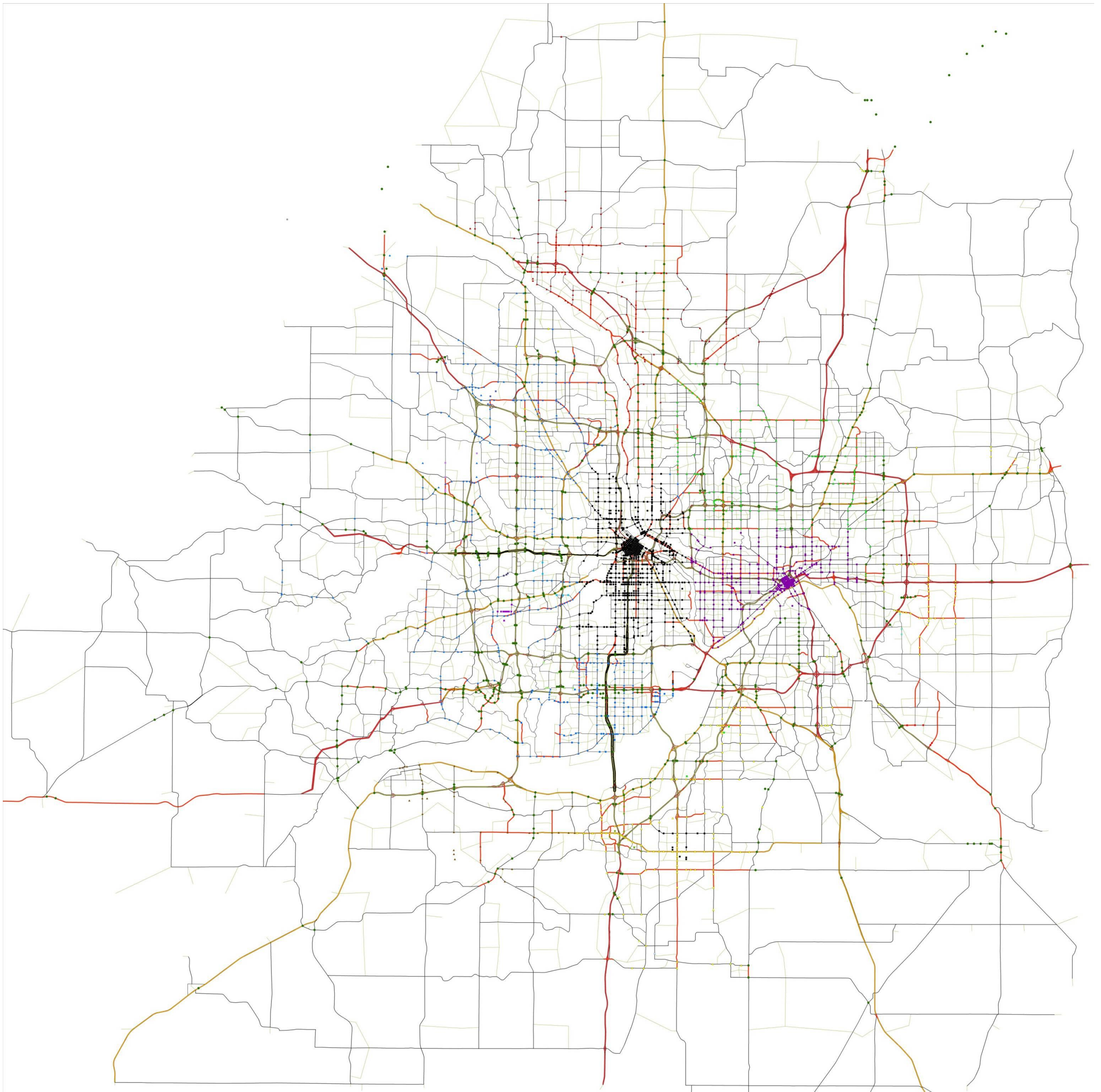
Figure 3.1 Twin Cities Regional Planning Network with Traffic Signals by Juristiction.

(If you are viewing this electronically, Zoom-in for details)

TABLE 3.1 Traffic Signal Locations by Government/Jurisdiction

Jurisdiction	Signals geo-located
ANOKA COUNTY	160
Apple Valley-County	12
Bloomington	53
Brooklyn Center	4
Brooklyn Park	6
Brooklyn Park/Champlin	1
Burnsville-County	18
CHASKA	1
Cottage Grove	5
DAKOTA COUNTY	142
Eagan-County	3
Eden Prairie	1
Edina	8
Golden Valley	6
HENNEPIN COUNTY	451
Hopkins	11
Jordan	1
Lakeville-County	1
MAC	3
Maple Grove	6
Minneapolis	803
Minnetonka	5
Mn/DOT	702
OAKDALE	2
Plymouth	7
RAMSEY COUNTY	182
Richfield	2
Saint Louis Park	6
Saint Paul	368
SCOTT COUNTY	43
WASHINGTON COUNTY	57
Wayzata	1
Woodbury	3

In addition to harvesting the traffic signal locations, traffic signal timing information data were also obtained. The level of success for obtaining this information varied considerably

across the jurisdictions contacted. For those that were able to provide timing data, many different format conventions were used to represent the timing information. For example Mn/DOT provided timing data in SYNCHRO 'zone' files. The zones represent a group of signals that were studied along a portion of one or more adjoining arterial corridors. Hennepin County provided manually constructed spreadsheets, one for each signal, while Scott County provided hand-written copies of timing data. Ramsey County provided scanned print-outs of their timing plans. Saint Paul also provided separate spreadsheets as well as paper copies for most of their signals. Minneapolis provided a spread sheet organizing all their controllers and control plans into a single table format. Other counties provided Aries compatible archives (specific to Econolite controllers). We were not successful in obtaining any signal timing data from Washington or Carver Counties. Weekday AM and PM peak plans which best matched the travel periods used in the SUE were used if and when available. Other than Minneapolis and Saint Paul, we were generally unsuccessful in obtaining timing data for traffic signals operated by municipalities. They did not have the resources to collect such information at the time of the request. A similar reason was given by staff at Washington County. Mn/DOT was not willing to release a large portion of their signal timing data due to security and privacy concerns of information within the data which could not easily be removed (for example, dial-in controller access modem phone numbers). The end result was that we harvested and then entered timing information for 2,519 signals. From the 3,074 that we geo-located only 2,631 signals were found to be located on an arterial link or node present in the Twin Cities Regional Planning (TCRP) model. In addition, signal associated with centroids, and centroid connector links were not included since such associations are problematic. Of over 300 signals which did not have signal timing entries, the majority (273) corresponded to the Mn/DOT signals. For the signalized intersections with missing timing information, a set of major/minor approach splits with 60/30 seconds were assumed. Approximately 81% of the signalized intersections with signal timing information were therefore used to estimate the arterial travel times. A summary of timing data categorized by jurisdiction that we were able to associate with the signalized intersection locations is given in Table 3.2.

TABLE 3.2 Traffic Signal Timing Entries by Government/Jurisdiction

Jurisdiction	Signal Timing Entries
ANOKA COUNTY	168
DAKOTA COUNTY	149
HENNEPIN COUNTY	459
MINNEAPOLIS	780
Mn/DOT	381
RAMSEY COUNTY	180
SAINT PAUL	367
SCOTT COUNTY	35

Further details regarding the format and processing of this information will be discussed in the final section of this chapter. The next section will describe the database structure and its implementation for estimating arterial link travel times within the network.

3.2 Database Structure

A database structure was developed to allow the travel time model to associate signalized intersection locations and attributes with TCRP model links. It is important to clarify at this point that in this project two individual versions of the TCRP were used in the SUE, the 1995 and the 2005 versions. The two models represent two snapshots of the TC roadway network as it evolved in time. The two versions have several differences mainly due to the following reasons:

- New road construction
- Increase of model density with the adition of more detailed link geometries
- Increase of model density with the inclution of additonal links and nodes.

Considering the existence of these two versions along with the need of facilitating a unified visualization medium for the A2D projects utilizing the travel times produced in this one, a common network was chosen to serve as base. This network is the 2009 conflated TCRP model which is the first version that has geometricaly correct links instead of describing the network as sticks-and-nodes. Another reason why the 2009 conflected network was used was that sufficient geo-spatial accuracy was needed to properly identify the traffic signals and associate them with the correct link or node. For the purpose of associating network elements in 1995/2005 with corresponding elements in 2009, two special correspondence tables were developed which 'mapped' links from either the 1995 or 2005 network to the equivalent link(s) in 2009. Several issues were encountered that rendered this procedure error prone and slow. For example, links existing in both 1995 and 2009 could not be associated because their IDs had been reassigned, the length of the links was changed during the model density increase introducing new nodes, and in several cases the geo-location of these links and nodes was significantly different between the two years.

The travel time algorithm was implimented as a closed, non-iterative procedure with the signal timing and warrant data information using the database schema presented in Figure 3.2. This is achieved by spatially joining geo-located intersection point records, each identified by SIG_ID, to the nearest link connection node identifier in the network model. First, in order to associate signalized intersections with the conflated network node points, a nearest neighbor, one-to-one spatial join was completed with GIS software. Second, in order to determine signalized intersections that lie on a link rather than a node, a spatial join intersecting the signalized intersection points was performed. The results of the spatial join are stored in two separate tables that associate the *LINK_ID* or *NODE_ID* attributes with a given *SIG_ID* attribute (not show in Figure 3.2; they are not included because they were not needed to implement the Skabardonis-Dowling model). A minimum distance tolerance search of 50 feet for either of these joins was used. Note that the spatial join also eliminated the traffic signals that were not associated with the conflated road network model. The remainder of this section will describe the generalized schema followed by pertinent details for tables developed by the Minnesota Traffic Observatory.
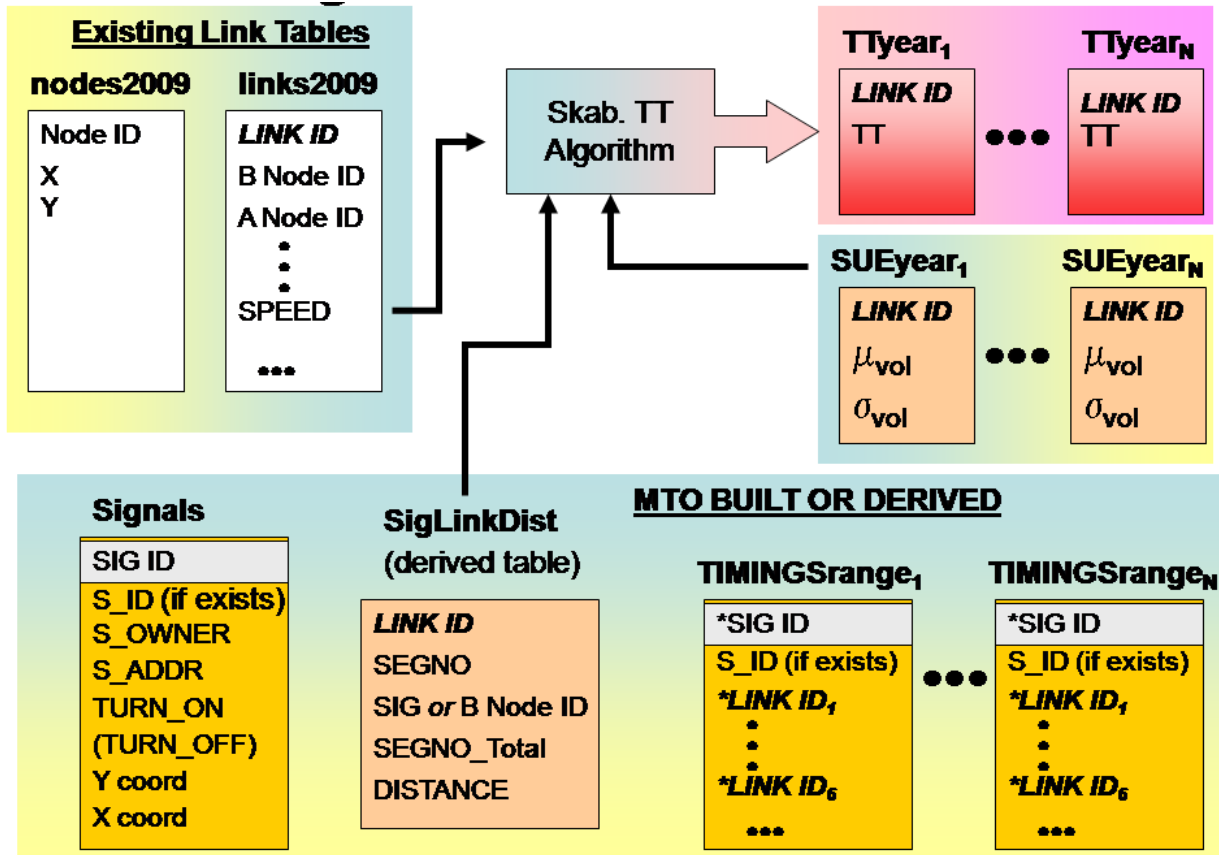
Figure 3.2 Signal Database Schema Overview.

The schema consists of several relational tables to support the algorithm implementation, and builds off of a geospatial database representation of the TCRP provided by the Met Council. The TCRP model database consists of two separate tables ("nodes2009" and "links2009" in Figure 3.2) that were conflated to nearest geographical locations of actual roadways which matched the road assignment group type for each link in the link table (attribute field not shown in Figure 3.2). The given record id for each link, LINK_ID, is used as a foreign key in many of the tables shown if Figure 3.2 to relate various attributes in the database. Second, the record id, , "SIG ID", for each signalized intersection is used for relating signal timing and other operations attributes. Refering to Figure 3.2, the signalized intersection locations and their timing information were constructed and organized manually (the Signals, and Timing<date range>1…n tables). The" SigLinkDist" table contains a list of all signals on the link or "B-node" end for each link. As opposed to being constructed manually, an algorithm was written to derive this table. Essentially, for each link the algorithm iterates through all links in the travel time model and computes cartesian distances between the beginning node point (A node) for each signal lying on the link up to its end-point (B node). A more detailed explanation of this table and its usage will be provided below.

The stochastic user equilibrium model (SUE) referenced earlier in this report was used to compute mean and variance traffic flow volume estimates for each of the links in the respective TCRP versions. These volumes were then corrected based on actual ATR counts, as described in earlier chapters of this report. For this study, four tables were derived from this process for the

years 1995 and 2005 and AM, PM Peak hours. With the help of the aforementioned correspondence tables these volumes were included in the 2009 TCRP database. The attributes and naming conventions of the MTO constructed or derived tables will be presented next. After the tables are described, a method to implement the Skabardonis-Dowling arterial travel time model will be presented.

3.2.1 Naming Conventions

The names of the tables reflect the time period being represented by the travel time calculations. For example, if the travel time output represents year=2005, the table name is "TT2005". Similarily, the derived traffic volume estimates for the same year is contained within a table name of "SUE2005". Signal timing plans are represented by a date range. At present there are no historical records of timing plans so the database will contain one table with a name "TIMINGS1900_2008". That is, any year between 1900 and 2008 will utilize the signal timing information from "TIMINGS1900_2008". As traffic signal timing plans change in the future and juristictions pass the changes to the MTO new tables can be constructed with a prescribed year range. The relevant attributes of the nodes2009 and links2009 tables from the met council, followed by the tables constructed and derived by the MTO will be described.

3.2.2 Nodes2009 Description

Each node point is represented by a pair of (X,Y) NAD83 UTM Zone15 coordinate attributes as well as a Node ID value. Node ID values are unique and represent either the beginning point (A) or end point (B), of a link.

3.2.3 Links2009 Description

Each link record in the network contains 24 attributes. Of interest to this project are 1. SPEED, the speed estimates at free-flow, 2. DISTANCE, the shape distance of the link segment in meters from A to B, 3. <AM/PM/OFF>CAP, the AM, PM, and off-peak capacities of the link, 4. ASGNGRP, the assignment group of the road, 5. A, the A-node point of the link, 6., B, the B-node of the link, and 7., LINK_FID, a unique key valued integer representing the link ID of the link record. Note that the capacity values were used to determine major-minor road designation to associate with the NEMA signal splits.

3.2.4 SigLinkDist Description

The utility for the derived SigLinkDist will be elucidated by the example provided next. The NODEID attribute stores either the signal id for the signalized intersection point location, or the B-node id for the link. The example illustrates 3 cases. The link id of 9013 represents a link whose B-node (end node) does not have a signal. Unlike link id 6039 as can be seen in Table 3.3 and Figure 3.3 for this example.
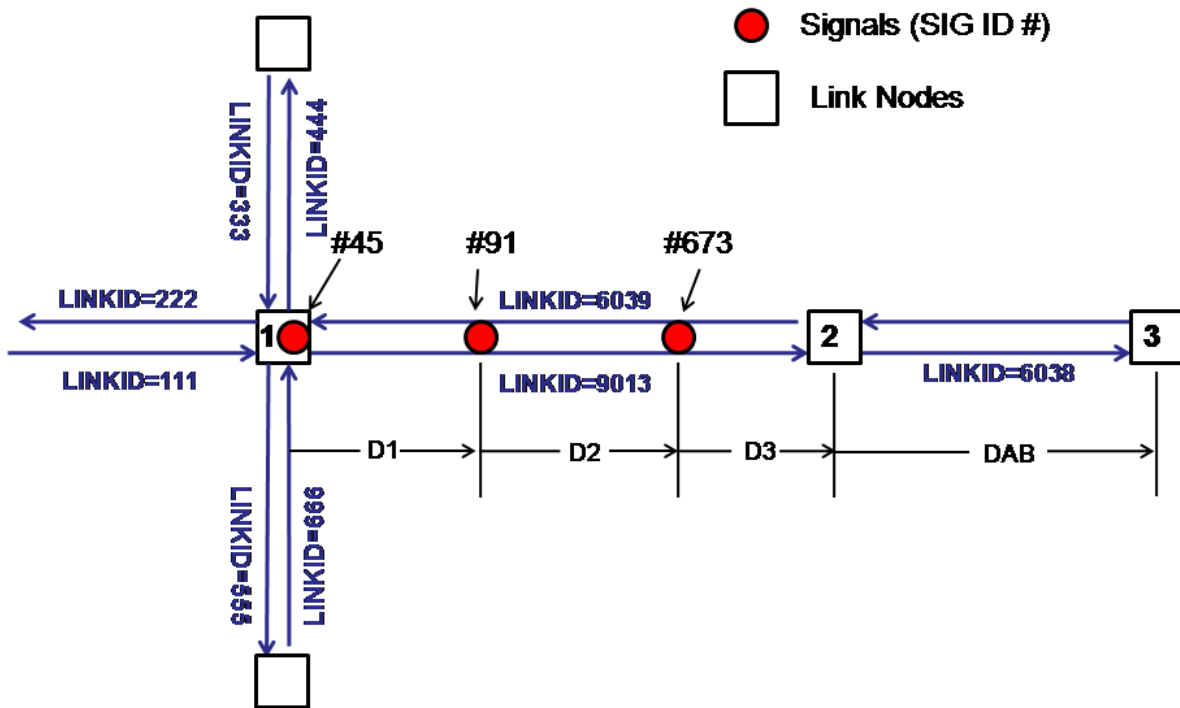
Figure 3.3 Link/node Example with Three Signalized Intersections.

TABLE 3.3 SigLinkDist Representation

FID_link	SEGNO	NODEID	SEGNO_TOTAL	DISTANCE
9013	1	91	3	D1
9013	2	673	3	D2
9013	3	**-2**	3	D3
6038	1	**-3**	1	DAB
6039	1	673	3	D3
6039	2	91	3	D2
6039	3	45	3	D1
…	…	…	…	…

Note that if there is no traffic signal present at the link's B node (links 9013, 6038), the NODEID < 0, and the value represents the B-node ID. For links with no signals, the distance is already calculated within the TCRP database and copied directly into the table for convenience. Table 3.4 list the attributes and their definitions for the "Signals" table in Figure 3.1.

3.2.5 TIMINGS<range> Description

Table 3.4 lists the fields utilized within the current database. The shaded attributes were derived by utilizing the A/B node coordinates, and capacity calculation attributes for each link. The NEMA splits were processed from manually tabulated signal timing data. The SIG_ID field

is derived by matching the S_ID and operating jurisdiction from the Signals table, with the same attributes contained within a table which stores the NEMA splits and cycle length. There were two such tables developed for this project, one for the AM peak, and one for the PM peak. The development of this table is explained in the Database Development Methods section.

What is important to note is the direct correspondance between ANGLEn , LINK_FIDn, and GSPLITn. Essentially, the correspondance creates an indirect reference to the link records in the TCRP database: the actual network Link ID is stored within LINK_FIDn. Its split value GSPLITn, is *referenced* to one of the NEMA phase splits, MAIN1_2/6, SIDE1_4/8 (EXTRA_1/2 was used for handling special cases) using the string name of the column. This allows the values of the green splits to be adjusted or modified independently, and separately from the links that are referencing the splits. The structure clearly deliniates the intent of split assignment to the given link. Using the previous example, the SIG_ID=45, with LINK_FID1=111, LINK_FID2=666, LINK_FID3=6039, LINK_FID4=333, LINK_FID5=*<NULL>*, LINK_FID6=*<NULL>*.

The numbers at the beginning of the CYCLE_LEN, MAIN1_2, represent the cycle length and green splits using the standard NEMA phase numbering convention. An implementation of the database to compute the arterial travel times will be presented next.

The Skabardonis-Dowling travel time model can then be implemented with the database as follows. Each of arterial links in the TCRP model are visited, utilizing the inter-distance calculations in SigLinkDist and speed estimates from the TCRP model to estimate the divided travel times across each link. The corrected volume attribute is utilized for the flow parameter. Each signal ID, SIG_ID, obtained from SigLinkDist table is used to key the proper signal timing record in the TIMING table. It also is used to compare the year with the TURN_ON date. Assuming the signal existed for the year to be examined, the split is obtained by searching which column n in LINK_FIDn contains the link ID record. Then the appropriate phase split is dereferencing from GSPLITn. The Cycle length is also obtained from CYCLE_LEN. As already noted, SigLinkDist reveals if no signalized intersection is contained for a given link as well, so for these cases the travel time based on the BPR model is estimated. The final travel time calculations are stored in the TTyear1.

Note the traffic volume estimates remain the same across the entire link. It is likely that the traffic volumes would vary across a link with signalized intersections lying within a link. However, the link model does not represent the cross streets at such intersections and are therefore not considered during the process.

TABLE 3.4 Signal Timing Attributes in the TIMINGS<range> Table

RECNUM	A unique identifier for each signal timing record
S_ID	An ID created by the operating jurisdiction, or by the MTO .
SIG_ID	Derived from matching S_ID and S_OWNER from the Signals table with S_ID and Operator from timing tables.
MAIN_NAME	The Main cross street name used in signal timing information data
SIDE_NAME	The Side cross street name used in signal timing information data
Operator	The jurisdiction that operates the signal, e.g., "Minneapolis", "MnDOT", "HENNEPIN COUNTY", etc. This is the same as the S_OWNER field.
ANGLE1	Direction of link 1, (0 degrees points East Bound)
ANGLE2	Direction of link 2, pointing in most opposite direction of Link 1.
ANGLE3	Direction of link 3
ANGLE4	Direction of link 4, pointing in most opposite direction of Link 3
ANGLE5	Direction of link 5
ANGLE6	Direction of link 6
LINK_FID1	The met council LINK_FID for link with ANGLE1
LINK_FID2	The met council LINK_FID for link with ANGLE2
LINK_FID3	The met council LINK_FID for link with ANGLE3
LINK_FID4	The met council LINK_FID for link with ANGLE4
LINK_FID5	The met council LINK_FID for link with ANGLE5
LINK_FID6	The met council LINK_FID for link with ANGLE6
GSPLIT_1	Green split ="MAIN1_X", SIDE1_X", …by LINK_FID1
GSPLIT_2	Green split ="MAIN1_X", SIDE1_X", …by LINK_FID2
GSPLIT_3	Green split ="MAIN1_X", SIDE1_X", …by LINK_FID3
GSPLIT_4	Green split ="MAIN1_X", SIDE1_X", …by LINK_FID4
GSPLIT_5	Green split ="EXTRA1_X", SIDE1_X", …by LINK_FID5
GSPLIT_6	Green split ="EXTRA1_X", SIDE1_X", …by LINK_FID6
CYCLE_LEN	Cycle Length
MAIN1_2	NEMA Green time for Main "East-bound" or "North-bound" phase
MAIN1_6	NEMA Green time for Main "West-bound" or "South-bound" phase
SIDE1_4	NEMA Green time for Side "South-bound" or "East-bound" phase
SIDE1_8	NEMA Green time for Side "North-bound" or "West-bound" phase
EXTRA_1	Note: only up to 4-legged intersections will be considered so far.
EXTRA_2	Note: only up to 4-legged intersections will be considered so far.

3.3 Database Development Method

There were several challenges that arose as the process to develop the database was carried out. Historical information was often missing, redundant, or conflicted with information obtained from a different agency for the same intersection. There are many reasons for this. Signal operations and maintenance agreements between different jurisdictions are renegotiated from time to time due to a myriad of circumstances. Some jurisdictions have integrated GIS into their signal and traffic operations and maintenance planning functions, however there is no common data representation. Furthermore, cross street address representations were, in many cases, incorrect, or were represented differently between signal timing data and the maintenance/operations inventory sheets provided by the various jurisdictions. This is because even 'correct' cross street addresses can, and were, identified under different nomenclature that reflect local, county, or state-aid provisions, as well as local knowledge of the vernacular used to refer to the road. The original goal of automating the data importation therefore became impractical and so we resorted to a systematic manual approach to address these problems.

The synopsis of building the database is organized into two parts. The first part summarizes the process used to geo-locate the signalized intersection locations with their warrant dates (*TURN_ON* attribute); the product of this effort was the Signals table shown in Figure 3.2. The second part summarizes processes used to enter and condense signal timing characteristics for each of the identified signalized intersection locations; the end product contained in the TIMINGS table in Figure 3.2. Both such processes, although organized and described separately, were commingled to resolve or uncover incorrect, missing, or redundant information. For example, signal timing data supplied by Mn/DOT actually provided several additional signalized intersection locations that were not contained within the original inventory list, or provided the 'missing' timing data not given by another jurisdiction. Under such circumstances, operating jurisdiction attribute was assigned according to who supplied timing data.

3.3.1 Part 1: Geocoding Method for Signalized Intersections

The first step was to geocode cross-street signal addresses obtained from each of the jurisdictions. This was accomplished utilizing Google Maps with a simple java script to extract the long-lat coordinates, i.e:

```
javascript:void(prompt('',gApplication.getMap().getCenter()));
```

Each cross-street address was entered with township and/or county designation. The signal location was further verified from available aerial and ground photos (www.lmic.state.mn.us or Google Maps & Google maps street view). A base map with the TLG 2005 center line roadmap overlaid with the conflated network was also used to query cross-street name addresses. When the warrant dates were not provided for a given signal, they were assigned a default year value of 1900. The geo-referenced signal location lists were then imported into the spatial database and then overlaid on top of the base map where the distances from the network link model were then compared to conflated 2009 network model. This process was used on cross-street address lists provided by Mn/DOT, Hennepin County, Washington County, Ramsey County, and Carver County.

Some jurisdictions – Dakota County, Anoka County, Scott County, Saint Paul, and Minneapolis -- provided GIS databases of their traffic signals as well as other traffic control

devices. Most of these data were corrected manually. Dakota and Scott counties required associating a cross-street address list with turn-on dates to the GIS locations. We also created a signal identifier key that was used to associate the timing data. Anoka County and Saint Paul also required associating turn-on date lists with the geo-referenced street addresses. In these cases, the jurisdictions already provided a unique signal identifier key to associate the separately provided signal timing data. Minneapolis required several cross-street address geocoded corrections, which were resolved utilizing the aforementioned geocoding method with Google maps.

Initially, we attempted to utilize linear referencing with the milepost data provided in the Mn/DOT list. The linear reference attributes appeared to reference 'route' GIS maps available on a Mn/DOT gis base map website (http://www.dot.state.mn.us/maps/gis base/html/datafiles.html). However, upon closer examination of the result from a linear referencing technique applied to the data, a large portion of the locations were of insufficient accuracy to be used for the project. This was the primary reason to revert to the aforementioned manual method.

After merging geocoded intersections together, redundant signals were removed. The technique to carry out this task was to assign a circular buffer area of 25 feet to each of the signals and test when signals fell within each signal's buffer area. A topology rule in ArcGIS was utilized to graphically identify the violations for visual inspection. Note that, as previously mentioned, for redundant signals reporting different operating jurisdictions we kept the traffic signal which had associated available timing data. We corroborated with the traffic engineers at the jurisdictions to resolve signalized intersection locations which were not able to properly geocode.

The conflated network provided a considerable improvement in geo-spatial accuracy compared to the other networks provided at the onset of the project. However, the conflated link representation of the roadways still do not perfectly overlay the TLG 2005 centerline map, and some of the link intersection representations were not conducive for implementing the Skabardonis-Dowling model. Figure 3.4 illustrates one of the more typical situations. Although the intersection contains four approaches, the conflation 'split' the intersection. The resolution was to duplicate the signal with a second signal on the opposite main-line link to ensure the delay factor for the signal is considered for both sides of the mainline. To conclude, future conflated networks will need to be checked carefully with the existing signalized intersection location data, with the locations within database adjusted accordingly to properly implement models that depend on the signals to be located accurately.
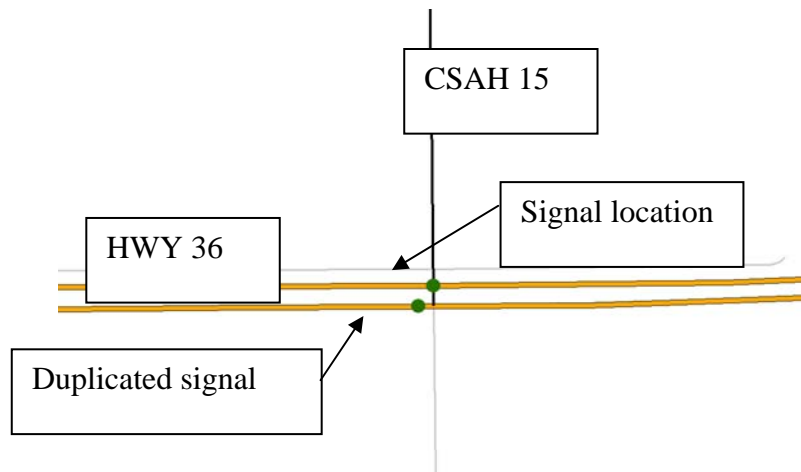
Figure 3.4 Geo-located Signalized Intersection Point Duplication.

3.3.2 Part 2: Method for Signal Timing Extraction and Representation

The Phase I arterial travel time model study utilized a single representative set of timing splits for each of two signalized intersections by averaging the manually observed timings in the field. This required an observer simultaneously stationed at each intersection.

The signal timing information therefore must similarly be reduced into an appropriate estimate of green/red timing splits for each approach direction that is meaningful and practical for the travel time model. The challenge of this step was complicated by the fact that signal timing is implemented several different ways and it is not practical to infer the measurement through manual observation with a network of this scale. A discussion of the approach is given next.

We harvested signal timing data from most of the counties, major townships and Mn/DOT. Not all jurisdictions utilize the same traffic signal controllers nor manage their timings in the same way. For example, Hennepin as well as Mn/DOT utilizes Econolite controllers. The city of Saint Paul deploys Safetran 170 controllers (Caltrans C1 platform), while Minneapolis uses a variety of controllers – including mechanical-analog and electro mechanical. Anoka County utilizes Econolite and Traconex controllers. As a result, signal timing data were provided in many different formats (paper, spreadsheet, software dependent-proprietary) with varying levels of completeness in their information, and representations of signal timing characteristics.

Some jurisdictions provided data in a digital, 'readable' format. Dakota County for example, provided data as an Aries archived signal timing database. The phase timing data for each intersection within defined zones in the Aries database are accessed one at a time, and subsequently converted to a 'readable' text file. A secondary post processing step was then developed to extract the minimum green, yellow, and all-red splits directly from the text file. Scott County provided hand-written paper copies.

The cities of Minneapolis and Saint Paul generally implement a varying number of fixed, fixed/group coordinated plans. For example, County and state operated traffic signals tend to be

semi-actuated (by the main street) or fully-actuated signals, in addition to being coordinated with a group master signal control in some instances. Protected turning movements are also common amongst many arterial intersections. The number of separate timing plans also differs considerably to reflect strategies used to reduce delay for time periods of the day.
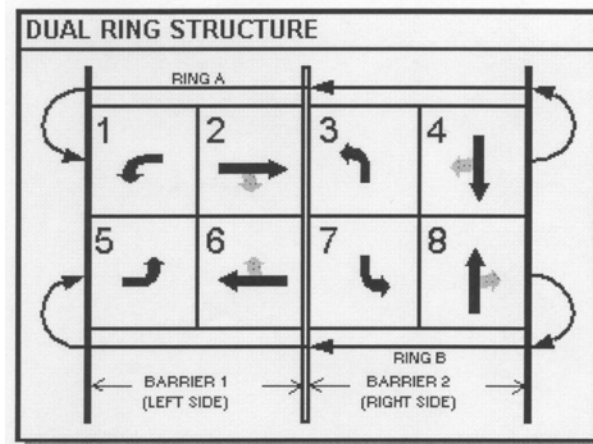

Figure 3.5 Standard NEMA Phase Assignments. The 'main' street is typically assigned to the left side barrier, here shown in the East-West-Bound convention (EWB).

Each jurisdiction follows a variation of the NEMA dual ring structure illustrated in figure 3.4. For example, the City of Saint Paul flips the NEMA convention upside down. Saint Paul utilizes twelve different conventions depending on the intersection approach geometry and number of phases utilized within the controller (RS170 type controllers). Others follow the diagram with a 'main-LEFT, minor-RIGHT ring barrier convention. For example, Ramsey county starts with phase 6 either NB or EB for the 'main road', and then proceeds clockwise around the intersection: HWY 96 runs EB (6), minor road, Hodgson SB (8), HWY 96 WB (2), and minor road, Hodgson NB (4). Minneapolis represented NEMA phases with twelve different conventions.

To conclude, because of the variety of format representations with which the intersection timing data were supplied, a largely manual process was utilized to tabulate most of the data into a unified container. Specifically, minimum green splits, yellow and all-red times for morning and afternoon timing plans which best correspond to the AM/PM peak hour periods used in the arterial travel time model were manually tabulated and then utilized as the timing attributes for each of the corresponding signalized intersections. The unified container was then post-processed to compute a cycle length and re-arrange the phases into the standard NEMA form illustrated in Figure 3.4. Further calculation of relative split and cycle lengths will be described below.

To address the fundamental questions for the regional arterial travel time model research, only a portion of the available signal timing data for the signalized intersections was utilized. In particular, actuated and semi-actuated signals—which are prevalent amongst several county and state operated traffic signals, do not have definitive green-time phase splits for their timing plans. For actuated control plans we utilized the following general formula in computing cycle length:

27

$$C = MIN(G_2 + Y_2, G_6 + Y_6) + MIN(G_4 + Y_4, G_8 + Y_8) + MAX(G_5 + Y_5, G_1 + Y_1) +$$
$$MAX(G_7 + Y_7 , G_3 + Y_3) + MAX(AR_7, AR_3) + MAX(AR_4, AR_8) + MAX(AR_2, AR_6) +$$
$$MAX(AR_5, AR_1)$$

where:

$MIN(x_1, x_2)$, $MAX(x_1, x_2)$ are the minima, maxima functions between x_1 and x_2
G_i is the minimum green or supplied green split for phase i.
Y_i is the all yellow split for phase i
R_i is the all red split for phase i.

The City of Saint Paul provided hourly averages of the green splits of the actuated timing plans, while Minneapolis provided single split values. No jurisdictions were able to provide historical records of past timing characteristics, and therefore the assumption was made that the timings were identical between the 1995 and 2005 years.

Lastly, once direction-convention is clearly established for each of the 8 possible phase splits, each of the NEMA splits must be related to the appropriate link direction in the network. Initially what was proposed for this step was to calculate and store the angle direction of each link as an additional link attribute, ANGLE*n* in Figure 3.2. Then the N links that connect to a given node containing a signalized intersection would be obtained as a relational query, along with their direction attributes to associate with the G_i/C timing splits. Since directional information for the approach phases within the supplied timing data was absent or ambiguous in many cases, a second approach was developed. The link capacity attributes were used to calculate average volume flows for each link. Then, the largest of the volume capacity calculation for each link pair in the met council network (for a 4-legged intersection for example) determined the links corresponding to each of the phase splits (Main 2/6 vs. side 4/8).

3.4 Travel Time Estimation Algorithm

The database structure and process described in the previous section were designed to be general and operational for any year and version of the TCRP model. For producing network wide travel times specifically for the years 1995 and 2005 slight modifications were used to capitalize on features in these two years of geometries and reduce the effort involved. Specifically, the algorithm as described in the previous sections requires that the volumes from the corrected SUE process be associated first with the appropriate links in the 2009 conflated TCRP database. In doing so, it is guaranteed that in any case where geometries have changed the algorithm will be the same. This of course implies that the table associating the two models has a perfect match for all 2009 links and all past year link volumes can be assigned in one or more of these links. In general, considering the possibilities in model evolution, it is possible to have many-to-many association, meaning that more than one link in 1995 are associated with a single link in 2009 and vice-versa. Fortunately, in the case of 1995 and 2005 only one-to-many relationships were found in arterial links mainly because 1995 geometry was refined in later versions of the model. Additionally, capitalizing on the fact that in the Skabardonis-Dowling model, if a link has more than one signal, the sum of red splits multiplied by 0.5(1-P) is the total control delay, where P (the proportion of vehicles arriving on green) is set equal to 0.48, which

was estimated using real travel time data collected from a sample of 50 arterial links in the Twin Cities metropolitan area during Phase I of this project. Due to both of the aforementioned characteristics we were able to combine the signal information with the 1995 and 2005 databases respectively, calculate the total delay due to the signal in each link, and with the combination of flow information estimate the travel time and average speed in each of the 1995 and 2005 links. The final step was to combine these results with links in the 2009 TCRP database with the help of the 2009 – 1995/2005 correspondence tables. Specifically, the link average speed in 1995/2005 is assumed the same in the corresponding link(s) in 2009 and based on their length a travel time is calculated.

The final products of this analysis are four databases containing the 2009 links along with the travel time a driver would experience if the conditions were those as in 1995 or 2005 (AM & PM). A flow chart of the algorithm finally used for this study can be seen in Figure 3.6.

3.5 Results

The goal of this phase was to utilize the theories and methodologies established previously to produce more accurate Travel Times on all Arterial links in the Twin Cities network for the years 1995 and 2005. Although the theoretical background in Phase I was relatively straightforward and the Skabardonis-Dowling equation is widely used, never have the above used in a task of this magnitude. The size of the network and resulting data introduced new problems that required engineering innovation to solve them. For the same reason it is difficult to present in a report the actual project results. In addition, this project produced arterial link travel times working in concert with another project that produced freeway and ramp travel times. The methodologies used there are fundamentally different from the ones used in the arterials simply because freeways and ramps are very well instrumented. Regardless, only the combined result is able to produce meaningful conclusions. Such conclusions regarding accessibility and the changes in accessibility by car in the Twin Cities metro area are the products of other projects fueled by the results produced here. Regardless, for the benefit of the reader we present the four summary figures

Figures 3.7 to 3.10, illustrate the AM and PM peak hour "distance in time" from/to all metro area TAZs to Minneapolis downtown in the years 1995 and 2005. Travel time in minutes ranged from 5 minutes in the vicinity of the downtown area to close to an hour for commutes starting at the outskirts of the metro area. It is interesting to note that in general travel times to downtown Minneapolis did not change dramatically between 1995 and 2005 and this modest change observed does not have a singular trend. Specifically, visible reductions are observed on origins near freeways. This can be attributed to the new ramp metering strategy implemented in 2003 that reduced considerably the ramp wait times. In contrast short trips from the west of the downtown area increased in travel time, most likely due to the increase of demand generated from the expansion of the west suburbs of the Twin Cities metro area. Similar travel time increases can be observed for trips originating on the northeast quadrant of the metro area.
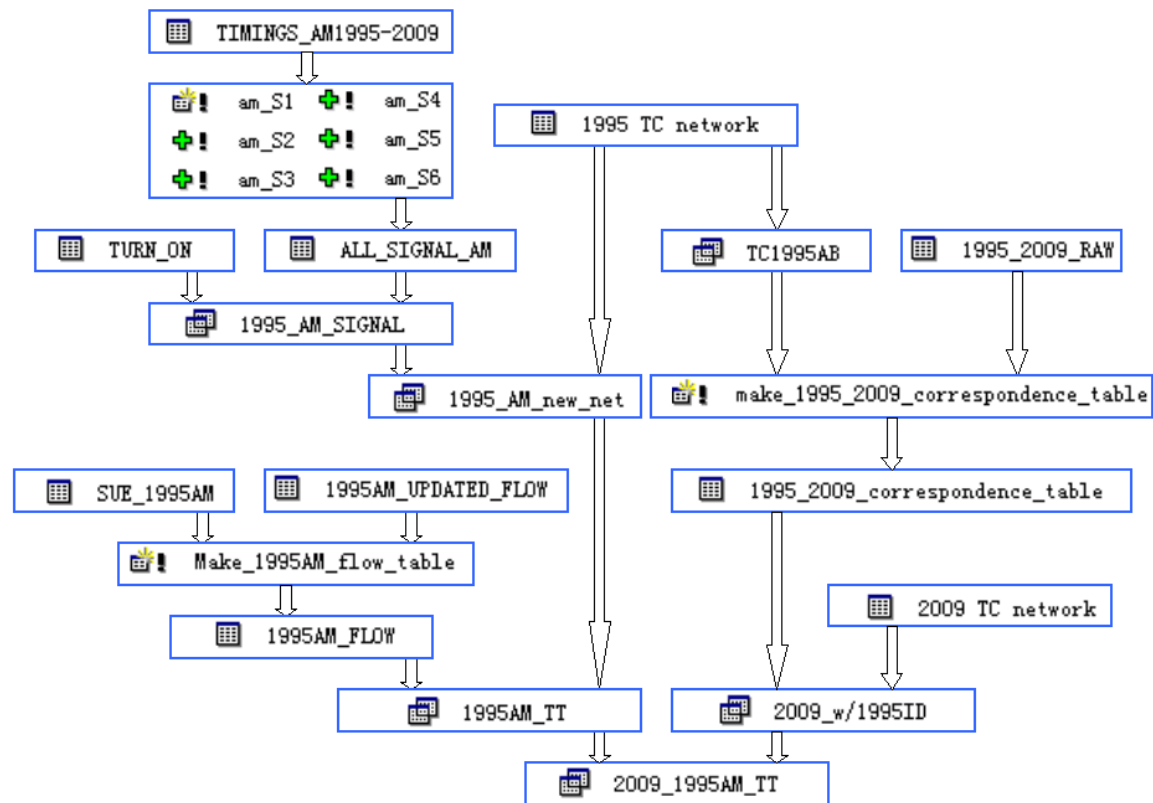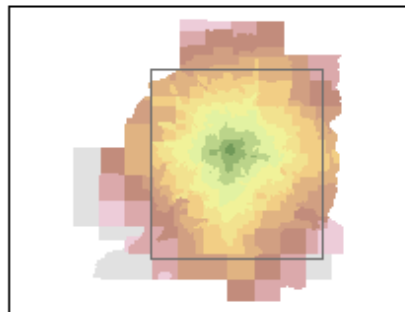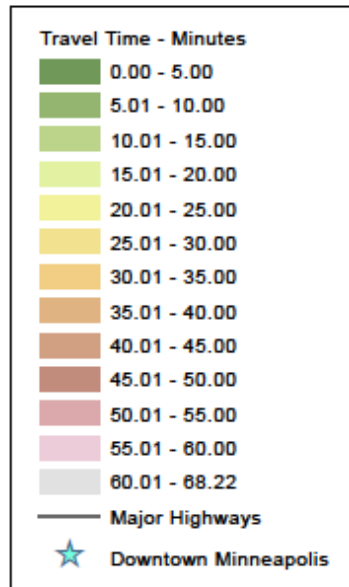
Figure 3.6 Travel Time Estimation Algorithm Flow Chart: 1995 AM Example.

Travel Time Contour Map
1995 AM Peak
Twin Cities, MN USA

Travel time by automobile to
DOWNTOWN MINNEAPOLIS

Travel Time - Minutes

	0.00 - 5.00
	5.01 - 10.00
	10.01 - 15.00
	15.01 - 20.00
	20.01 - 25.00
	25.01 - 30.00
	30.01 - 35.00
	35.01 - 40.00
	40.01 - 45.00
	45.01 - 50.00
	50.01 - 55.00
	55.01 - 60.00
	60.01 - 68.22
——	Major Highways
★	Downtown Minneapolis

Zone Structure Displayed: Traffic Analysis Zone Boundaries (TAZ)
Primary Data Sources: MnDOT, Twin Cities Metropolitan Council,
US Census Bureau 1990 & 2000, and the NEXUS Research Group
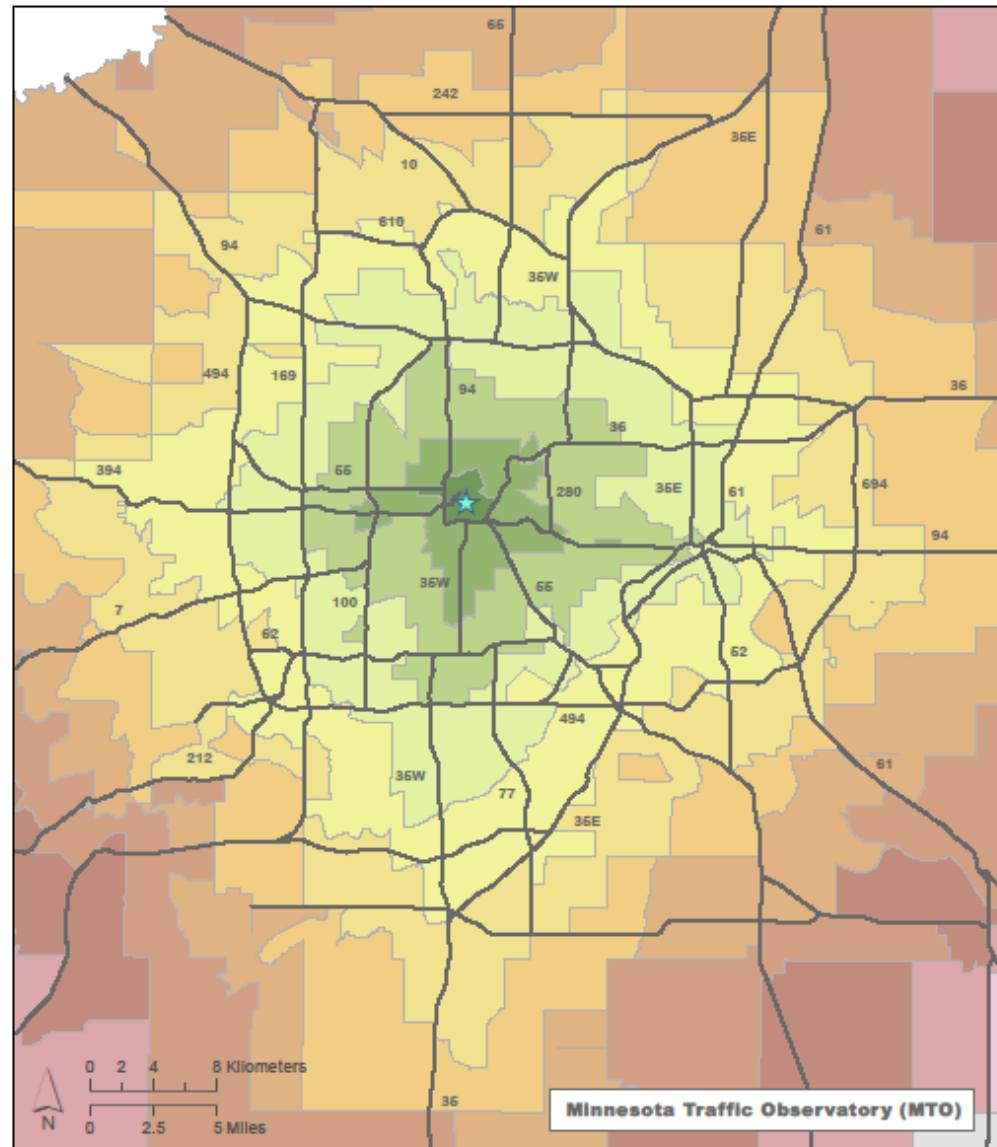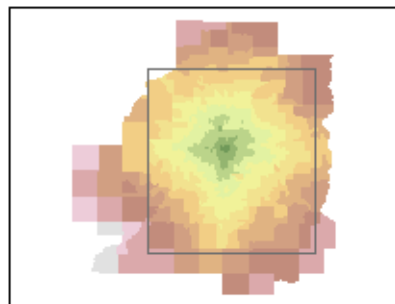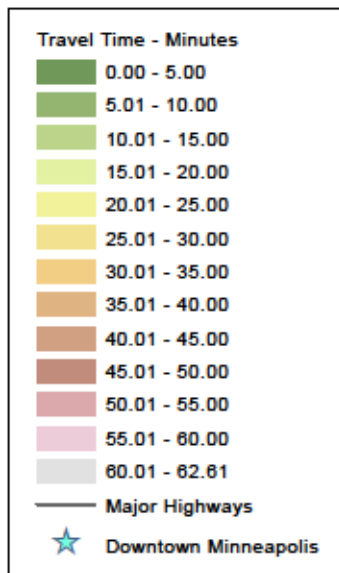Map Author: the Minnesota Traffic Observatory (MTO)

Figure 3.7 1995 Travel Time by Automobile to Downtown Minneapolis, AM Peak Hour.

Figure 3.8 2005 Travel Time by Automobile to Downtown Minneapolis, AM Peak Hour.

Figure 3.9 1995 Travel Time by Automobile to Downtown Minneapolis, PM Peak Hour.

Travel Time Contour Map
2005 AM Peak
Twin Cities, MN USA

Travel time by automobile to
DOWNTOWN MINNEAPOLIS

Travel Time - Minutes

	0.00 - 5.00
	5.01 - 10.00
	10.01 - 15.00
	15.01 - 20.00
	20.01 - 25.00
	25.01 - 30.00
	30.01 - 35.00
	35.01 - 40.00
	40.01 - 45.00
	45.01 - 50.00
	50.01 - 55.00
	55.01 - 60.00
	60.01 - 66.11

—— Major Highways

☆ Downtown Minneapolis

Zone Structure Displayed: Traffic Analysis Zone Boundaries (TAZ)
Primary Data Sources: MnDOT, Twin Cities Metropolitan Council,
US Census Bureau 1990 & 2000, and the NEXUS Research Group
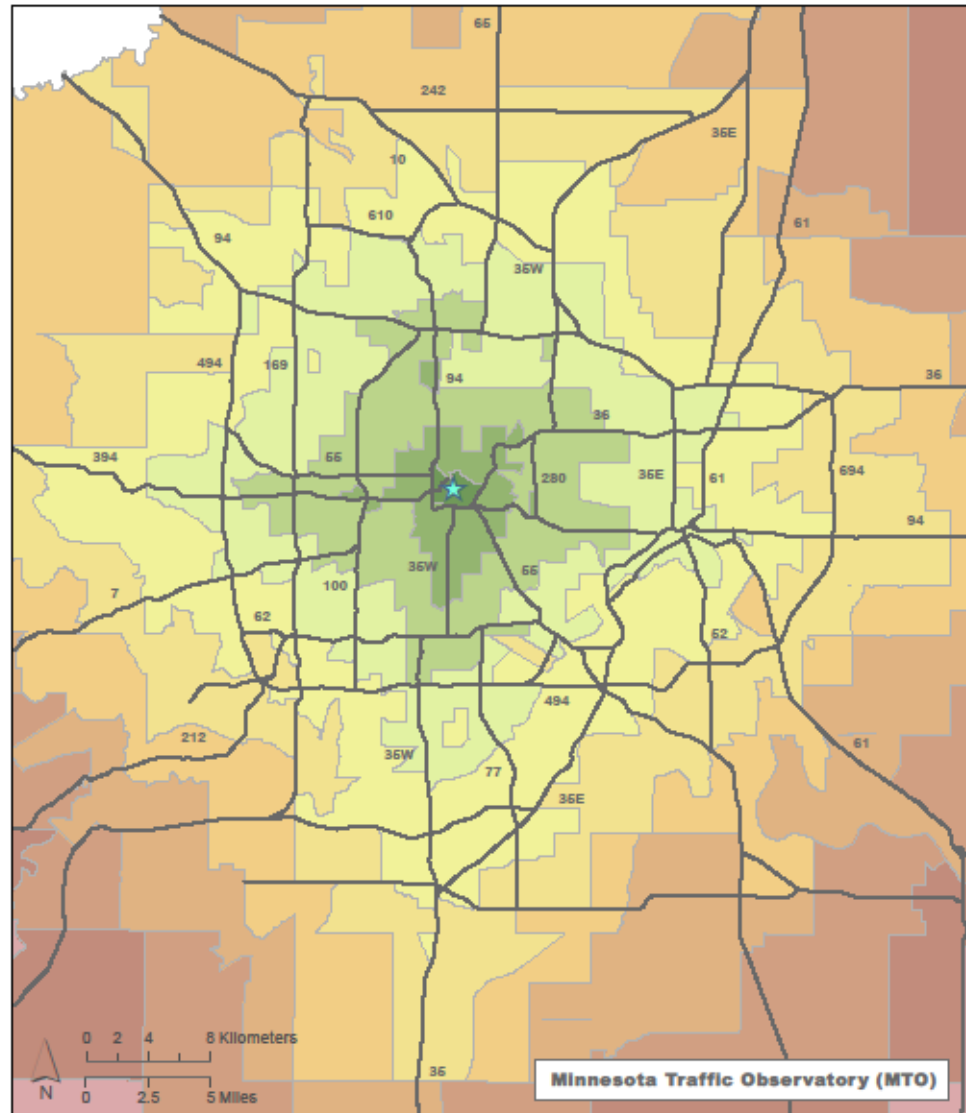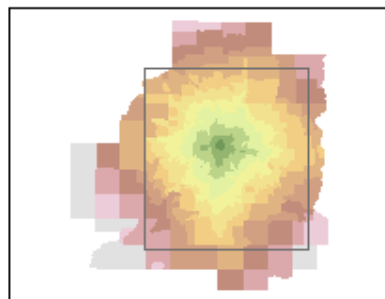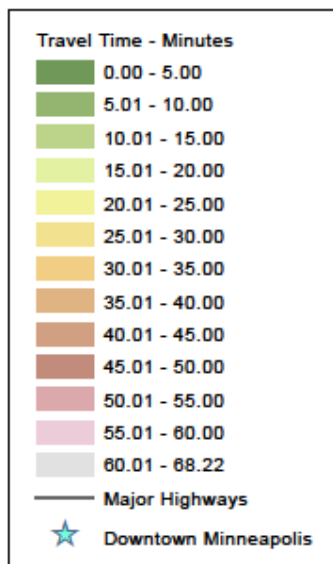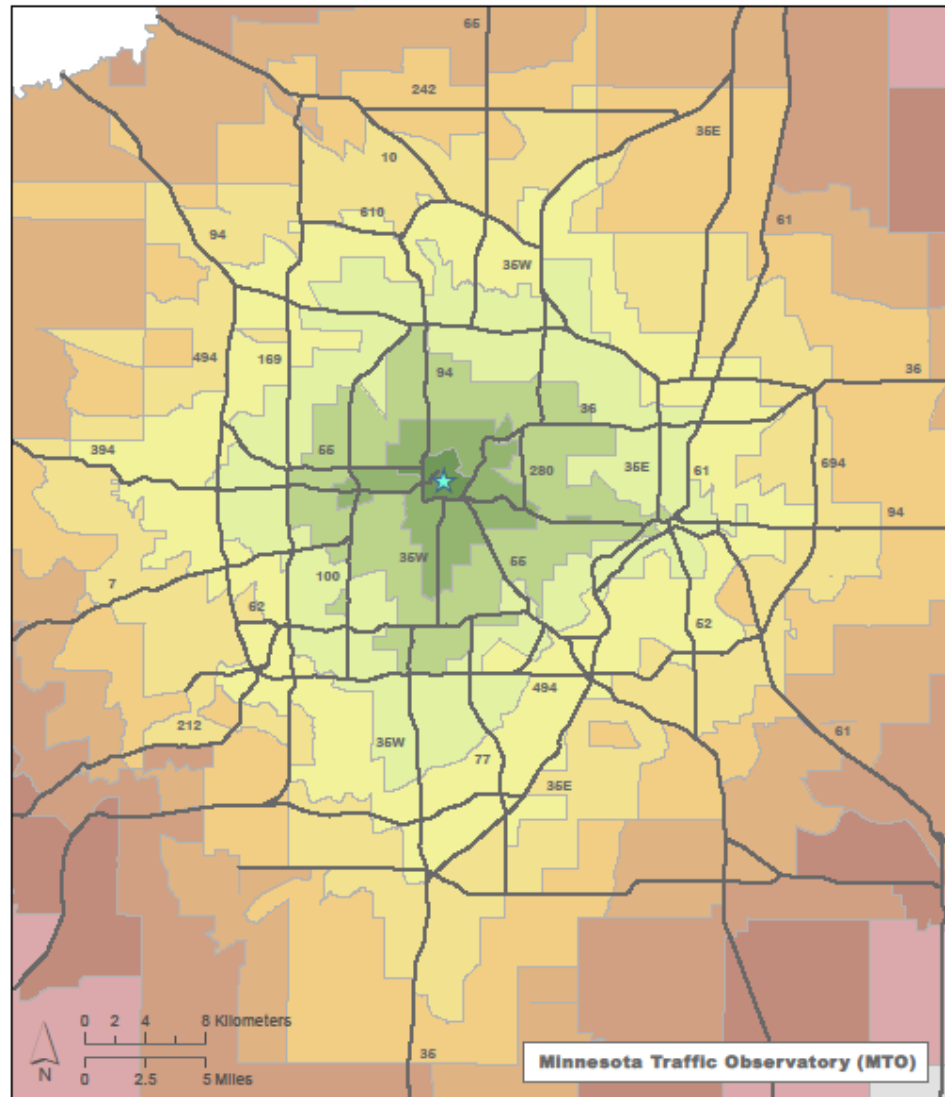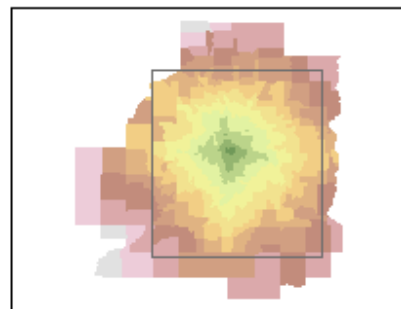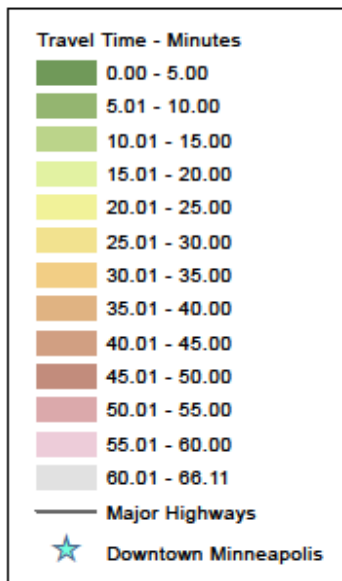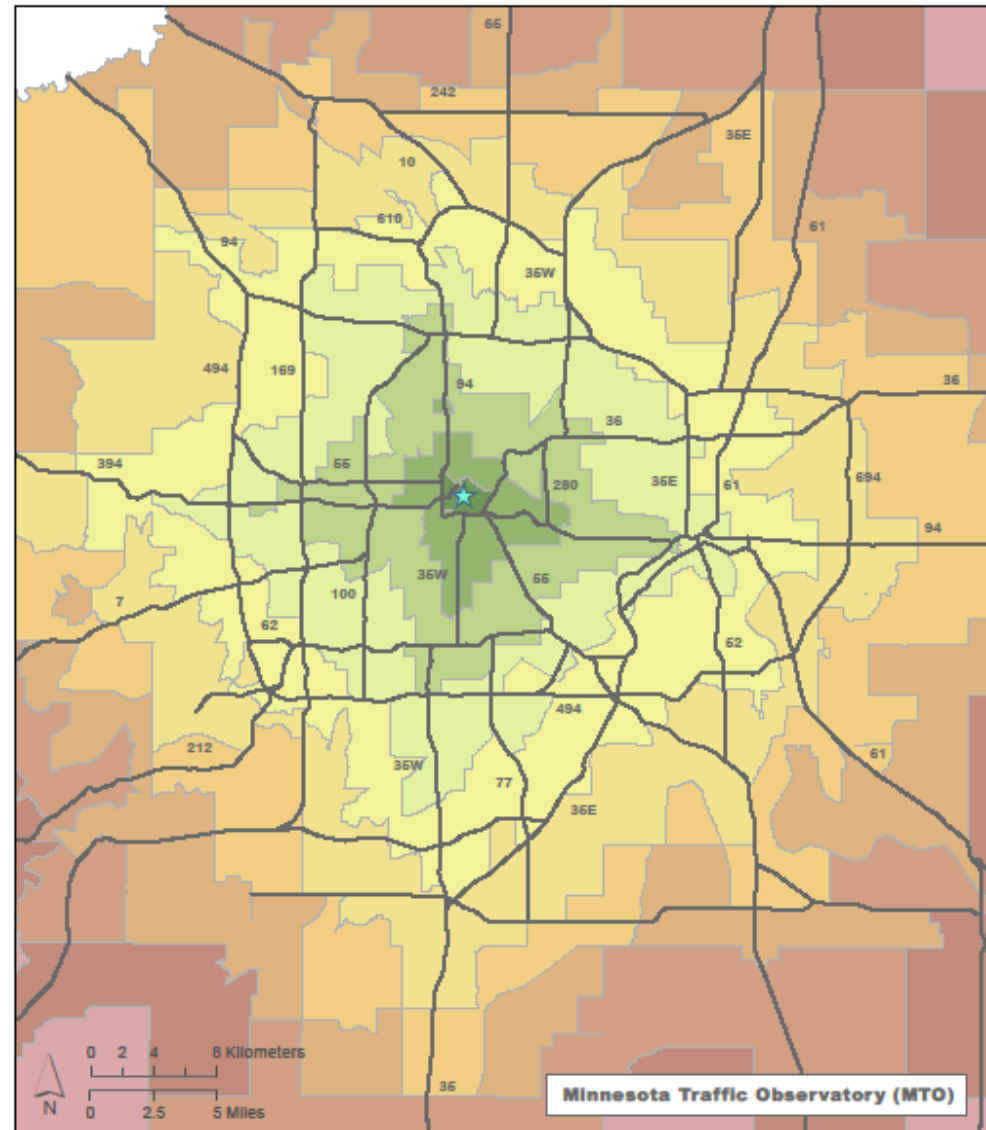Map Author: the Minnesota Traffic Observatory (MTO)

Figure 3.10 2005 Travel Time by Automobile to Downtown Minneapolis, PM Peak Hour.

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Chapter 4: Summary and Conclusions

As noted in Chapter 1, the objectives of this project were to (a) produce historic estimates of travel times on Twin-Cities arterials for 1995 and 2005, and (b) develop an initial architecture and database that could, in the future, produce timely estimates of arterial traffic volume and travel time. Estimating traffic volumes and travel times is relatively straightforward on roads with a spatially dense placement of traffic sensors, but becomes more difficult where sensors are sparse or nonexistent. Our Phase I field study indicated though that on arterial links where both the demand traffic volume and the signal timing are known, model-based estimates of travel time that are on average within 10% of measured values can be obtained. Phase II of this project then focused on an initial application of this approach to the entire Twin Cities arterial system. The Phase II effort divided into three main subtasks: (1) updating estimates of demand traffic volume obtained from a transportation planning model to make them consistent with available volume measurements, (2) compiling information on traffic signal locations in the Twin Cities and compiling this into a geographic database, and (3) combining the updated traffic volumes and signal information to produce link-by-link peak-period travel time estimates on Twin Cities arterials for 1995 and 2005.

Although we were able to produce useable estimates for the Access to Destinations program, the potential of this approach for ongoing monitoring of arterial volumes and travel times was not realized as fully as it could have been. In large part this was due to the need to work within historic data limitations and accommodate historic representations of the arterial network. That is, in order to produce methodologically similar estimates of traffic volume for 1995 and 2005 we were limited to arterial data that were available for both time periods, essentially the Mn/DOT arterial ATRs. In addition, earlier versions of the Twin Cities planning model often contained generalized representations of the road system, where a single generalized link was used to represent either parallel streets in a corridor, or several signalized intersections in series. A substantial effort was thus required to establish correspondences between the 1995 and 2005 planning networks and geographically more faithful databases. For future monitoring of arterial conditions it should be possible to make use of increasingly more plentiful sources of traffic data, combined with more accurate representations of the road system. In the remainder of this chapter we would like to suggest some additional steps that could bring this about.

First, improve and enhance the traffic volume updating methods. As indicated earlier, accurate model-based estimation of arterial travel times requires accurate estimates of traffic volume, and for future monitoring of arterial traffic conditions it is desirable to use additional data sources when/if they become available. In particular, enhancements that allow effective use of short-count data would be especially helpful. Since short counts provide potentially biased estimates of AADT, rather than the actual measurements of AADT as provided by ATRs, the volume updating procedures will have to be modified to allow for this uncertainty, but this is technically feasible.

In addition, as currently implemented, the volume updating procedures do not attempt to update origin-destination information. In principle though it is possible to extend the Gaussian

process framework to provide updates of the origin-destination matrix as well as the unobserved link volumes. The covariance of link i and link j (Davis and Nihan 1993; Davis 1994) is

$$Q_{ij} = \sum_k \bar{d}_k (P_{ijk} - P_{ik}P_{jk}) + \sum_k \sigma_k^2 P_{ik}P_{jk} = \left(\widetilde{Q}(\bar{d}) + HV_0 H^T \right)_{ij} \qquad (4.1)$$

where $\bar{d}_k$ is the kth demand in the OD table;

P_{ijk} is the probability that both links i and j are used;

P_{ik} is the probability that link i is used;

H is the link use probability matrix; and

V_0 is the OD demand covariance with diagonal element equal to σ_k^2 and off-diagonal elements equal to 0.

The observed/unobserved link flows and OD demands approximately follow a multinomial distribution (Maher 1983). If observations are error-free, they have the form:

$$\begin{bmatrix} X_A \\ X_U \\ d \end{bmatrix} \sim N \left[\begin{pmatrix} \mu_A \\ \mu_U \\ \bar{d} \end{pmatrix}, \begin{pmatrix} \widetilde{Q}_{AA}(\bar{d}) + H_A V_0 H_A^T & \widetilde{Q}_{AU}(\bar{d}) + H_A V_0 H_U^T & H_A V_0 \\ \widetilde{Q}_{UA}(\bar{d}) + H_U V_0 H_A^T & \widetilde{Q}_{UU}(\bar{d}) + H_U V_0 H_U^T & H_U V_0 \\ (H_A V_0)^T & (H_U V_0)^T & V_0 \end{pmatrix} \right] \qquad (4.2)$$

The mean of the updated OD vector is given by

$$E(d \mid X_A) = \bar{d} + (H_A V_0)^T (\widetilde{Q}_{AA}(\bar{d}) + H_A V_0 H_A^T)^{-1} (X_A - H_A \bar{d}) \qquad (4.3)$$

This suggests that freeway detector data, although of limited value for directly updating arterial volumes, could have an indirect effect by supporting improved estimates of travel demand.

Second, complete the signal timing database. This work was not completed in phase II due to the unforeseen difficulties encountered concerning the availability and accuracy of the information. Specifically, one timing plan, operating at the time the information was retrieved (2008) was used for each intersection. Timing plans change over the period of the day to accommodate Morning and Afternoon peak period traffic patterns, as well as approximately every two years, the average period where each jurisdiction retimes all controlled intersection. This effort would involve, for the signals we currently have historical information and/or multiple daily plans, conclude the importation of these signal timing information into the GIS database already developed.

Third, integrate the signal information into the SUE model. Until now the signal information was only utilized at the end of the process that combined them with expected

volumes to compute link travel times. It is more appropriate for this information to be integrated with the SUE traffic assignment model that generates the relevant link volumes.

Fourth, design an Arterial Travel Time Factory (ATTF): The current process for producing link travel times is cumbersome, separated into many individual steps, and requires specialized knowledge of the software and algorithms used. Such characteristics are counterproductive when the goal is to replicate this procedure many times in the production of arterial link travel times for various times of day, either historically or for the present. This effort would streamline the process of utilizing the developed procedures in the production of travel times while ensuring that this procedure can be performed by available Minnesota Traffic Observatory personnel, so our ability to do so will not be harmed with the departure of the researchers involved in its development. This can be considered as a technology transfer task, moving research results into engineering practice.

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Appendix A: Calculation of the Link Flow Covariance Matrix

Calculation of the Link Flow Covariance Matrix

The O-D demands are treated as Poisson random variables. The link flow covariance between links i and j is then given by:

$$Q_{ij} = \begin{cases} \sum_k d_k P_{ijk} & (i \neq j) \\ \sum_k d_k P_{ik} & (i = j) \end{cases} \qquad\qquad \text{(A.1)}$$

where d_k is the kth demand in the O-D table;

P_{ijk} is the probability that both links i and j are used; and

P_{ik} is the probability that link i is used.

Let I_1, I_2 (J_1, J_2) denote the start node and the end node of link i (j). For simplicity, we will ignore subscript k for O, D, and W (number of effective routes based on the shortest path calculation of origin O). For a given O-D pair, a route using both link i and link j could be either $O \to I_1 \to I_2 \to J_1 \to J_2 \to D$ or $O \to J_1 \to J_2 \to I_1 \to I_2 \to D$. So the probability of using both links takes the form

$$P_{ijk} = \frac{W_{O \to I_1} \times W_{I_1 \to I_2} \times W_{I_2 \to J_1} \times W_{J_1 \to J_2} \times W_{J_2 \to D}}{W_{O \to D}} + \frac{W_{O \to J_1} \times W_{J_1 \to J_2} \times W_{J_2 \to I_1} \times W_{I_1 \to I_2} \times W_{I_2 \to D}}{W_{O \to D}} \quad \text{(A.2)}$$

In Eq. (A.2), the weighting terms Ws could be calculated by the modified Dial's forward pass algorithm as follows (using $W_{A \to B}$ as an example):

Step 1. Perform the one-to-all shortest path algorithm for current origin O and calculate the link likelihood.

Step 2. Set node weight of A equal to 1 and others' equal to 0.

Step 3. Update the node weight of the next nearest node from A.

Step 4. Repeat step 3 until B is reached.

Step 5. Return the node weight of B ($W_{A \to B}$).

If directly applying the above-mentioned modified Dial's algorithm to calculate Q_{ij} for each O-D pair, it will produce numerous redundancies and be extremely time consuming. Three tricks were used to speed up the Q calculation.

Trick 1. The one-to-all shortest path only needs to be executed once for each origin O.

Van Vliet (1980) showed that for a given O-D pair,

$$\sum_R \varepsilon_R = W_{O \to D} \qquad (\varepsilon_R = e^{-\theta(C_R - C^*)}) \qquad\qquad \text{(A.3)}$$

where ε_R = effectiveness of route R;

C_R = cost of route R;

C^* = minimum cost of route R;

$C_R - C^*$ = excess route cost; and

θ = perception parameter.

Note that the excess cost of route $O \rightarrow I_1 \rightarrow I_2 \rightarrow J_1 \rightarrow J_2 \rightarrow D$ is the sum of excess route costs along it, i.e.

$$\Gamma_{O \rightarrow I_1 \rightarrow I_2 \rightarrow J_1 \rightarrow J_2 \rightarrow D} = \Gamma_{O \rightarrow I_1} + \Gamma_{I_1 \rightarrow I_2} + \Gamma_{I_2 \rightarrow J_1} + \Gamma_{J_1 \rightarrow J_2} + \Gamma_{J_2 \rightarrow D} \qquad (A.4)$$

where Γ = excess route cost.

By the additive property for exponential terms, Eq. (A.2) is expressed in a form of weights not effectiveness of routes. However, Eq. (A.3) is true only when both the links i and link j are on the feasible path(s) of O-D, meaning the minimum costs of start and end nodes for each link must be measured from the origin O.

Trick 2. The node-to-node weight matrix WNTN (number of effective route matrix) should be calculated for each origin O.

Instead of finding $W_{O \rightarrow I_1}$, $W_{O \rightarrow J_1}$, $W_{O \rightarrow D}$, $W_{I_1 \rightarrow I_2}$, $W_{J_1 \rightarrow J_2}$, $W_{I_2 \rightarrow J_1}$, $W_{J_2 \rightarrow I_1}$, $W_{I_2 \rightarrow D}$, and $W_{J_2 \rightarrow D}$ separately, we calculate WNTN for each origin which can be considered as "one stop" station, which allows us simply "look up" to obtain all node weights for Q_{ij}. Although many of the elements WNTN will not be used, it is much faster than identifying and saving the information about which are the current links to calculate weights, and which are not.

Trick 3. For each origin O, all destination-irrelevant weights should be checked first; then if needed, calculate the destination-related terms and sum them up.

Rewriting Eq. (A.2) gives

$$P_{ijk} = W_{I_1 \rightarrow I_2} \times W_{J_1 \rightarrow J_2} \times \left[W_{O \rightarrow I_1} \times W_{I_2 \rightarrow J_1} \times \frac{W_{J_2 \rightarrow D}}{W_{O \rightarrow D}} + W_{O \rightarrow J_1} \times W_{J_2 \rightarrow I_1} \times \frac{W_{I_2 \rightarrow D}}{W_{O \rightarrow D}} \right] \quad (A.5)$$

In Eq. (A.5), (1) $W_{I_1 \rightarrow I_2}$, $W_{J_1 \rightarrow J_2}$ are link likelihoods. If either of them is 0, P_{ijk} is 0. it is not necessary to do further computation for this link pair. (2) Since routes $O \rightarrow I_1 \rightarrow I_2 \rightarrow J_1 \rightarrow J_2 \rightarrow D$ and $O \rightarrow J_1 \rightarrow J_2 \rightarrow I_1 \rightarrow I_2 \rightarrow D$ cannot be both feasible, so we calculate $W_{I_2 \rightarrow J_1}, W_{J_2 \rightarrow I_1}$ to determine which route is. (3) Only $\frac{W_{J_2 \rightarrow D}}{W_{O \rightarrow D}}$ and $\frac{W_{I_2 \rightarrow D}}{W_{O \rightarrow D}}$ depend on D, which reminds us factoring other terms out and sum up one of them first. Now Eq. (A.1) becomes

$$Q_{ij} = \begin{cases} \sum_O W_{I_1 \rightarrow I_2} W_{J_1 \rightarrow J_2} \left[W_{O \rightarrow I_1} W_{I_2 \rightarrow J_1} \sum_D d_{O \rightarrow D} \frac{W_{J_2 \rightarrow D}}{W_{O \rightarrow D}} + W_{O \rightarrow J_1} W_{J_2 \rightarrow I_1} \sum_D d_{O \rightarrow D} \frac{W_{I_2 \rightarrow D}}{W_{O \rightarrow D}} \right] & (i \neq j) \\[4ex] \text{SUE flow of Link } i & (i = j) \end{cases}$$